Published by Institute of Physics Publishing for SISSA

Received: *May 13, 2005*
Revised: *September 2, 2005*
Accepted: *October 21, 2005*
Published: *November 15, 2005*

Electroweak evolution equations

Paolo Ciafaloni

INFN - Sezione di Lecce
Via per Arnesano, I-73100 Lecce, Italy
E-mail: paolo.ciafaloni@le.infn.it

Denis Comelli

INFN - Sezione di Ferrara
Via Paradiso 12, I-35131 Ferrara, Italy
E-mail: comelli@fe.infn.it

Abstract: Enlarging a previous analysis, where only fermions and transverse gauge bosons were taken into account, we write down infrared-collinear evolution equations for the Standard Model of electroweak interactions computing the full set of splitting functions. Due to the presence of double logs which are characteristic of electroweak interactions (Bloch-Nordsieck violation), new infrared singular splitting functions have to be introduced. We also include corrections related to the third generation Yukawa couplings.

Keywords: Parton Model, Standard Model, NLO Computations.

JHEP11(2005)022

© SISSA 2005

http://jhep.sissa.it/archive/papers/jhep112005022/jhep112005022.pdf

Contents

1. Introduction **1**

2. A working example: left fermions in the $g' \to 0$ limit **3**

3. Full EW evolution equations **8**

A. Kernels of the evolution equations **16**

B. Fermion-scalar sector Yukawa interactions **17**

C. States classification **17**

JHEP11(2005)022

1. Introduction

Energy-growing electroweak corrections in the Standard Model have received recently a lot of attention in the literature, being relevant for LHC physics [1], for Next generation of Linear Colliders (NLCs) [2] and for ultrahigh energy cosmic rays [3]. The presence of double logs ($\log^2 \frac{s}{M^2}$ where M is the weak scale) in one loop electroweak corrections has been noticed in [4]. One loop effects are typically of the order of 10-20 % at the energy scale of 1 TeV, so that the subject of higher orders and/or resummation of large logarithms has to be addressed. After the observation [5] that double and single logs that appear in the one loop expressions are tied to the infrared structure of the theory, all order resummation has been considered at various levels for exclusive processes: Leading Log (LL) [6], Next to Leading Log (NLL) [7] and so on. Moreover many fixed order analyses at the one [8] and two loop [9] level have been performed.

Collinear evolution equations are written in general with the purpose of resuming large contributions of logarithmic type by factorizing collinear singularities. This is done by separating a "soft" scale contribution that includes collinear logarithms and a "hard" scale contribution which is free of logarithms and therefore perturbative. However in the electroweak sector at energies $\sqrt{s}$ much higher than the weak scale $M \sim M_Z \sim M_W \approx 100\,\mathrm{GeV}$, logarithms of both collinear *and* infrared origin appear, even in fully inclusive quantities [10]. In a previous paper [11] we have written infrared evolution equations for fully inclusive observables (i.e. including W and Z radiation) in the limit of vanishing U(1) coupling and considering only fermions and transverse gauge bosons, showing that the presence of both

infrared and collinear singularities can be tackled with by introducing new, infrared singular, splitting functions. The aim of this work is to complete the analysis by writing down infrared evolution evolution equations in the full $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ electroweak sector of the Standard Model, including left and right handed fermions, transverse and longitudinal bosons degrees of freedom. Since the third family Yukawa couplings are non negligible, we include their contribution; on the other hand we neglect family mixing effects related to the Cabibbo Kobayashi Maskawa matrix. A complete analysis of collinear singularities in the Standard Model will also have to include QED evolution equations [12] for transverse momenta of the emitted particles lower than the weak scale M, and QCD DGLAP [13] evolution equations.

The inclusiveness of our observables allows to neglect final state radiation (see ref. [18] for details), contrary to what happens in the case of exclusive observables, and allows to single out features that are peculiar of the initial states.

The analysis of mass singularities in a spontaneously broken gauge theory like the electroweak sector of the Standard Model has many interesting features. To begin with, initial states like electrons and protons carry nonabelian (isospin) charges; this feature causes the very existence of double logs i.e. the lack of cancellations of virtual corrections with real emission in inclusive observables [10]. Secondly, initial states that are mass eigenstates are not necessarily gauge eigenstates; this causes some interesting mixing phenomena analyzed in [14 – 16]. Technically speaking, a complication is due to the fact that the gauge theory Ward Identities are broken by Goldstone boson contributions; however these contributions are proportional to the symmetry breaking scale and are therefore expected to be suppressed to a certain extent in the high energy limit. These contributions have been shown to be negligible at the one loop level [17, 11]; we assume this to be true at higher orders as well, although we lack a formal proof at the moment. Finally, collinear factorization has been proved at the one loop level [17]; we assume it to be valid to all orders. Ultraviolet logarithms leading to running coupling effects are neglected throughout the paper.

The main results of this paper are contained in eqs. (3.1)–(3.7), that represent the $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ electroweak evolution equations for the full SM spectrum. Namely, eqs. (3.1) are written for matricial structure functions $\underset{j}{\overset{i}{\mathcal{F}}}{}^{\alpha\beta}_{\alpha'\beta'}$ in isospin space (see figure 1). In order to make these equations useful for practical purposes, we have to write them for corresponding scalar quantities. This is done by exploiting the (recovered) isospin and CP symmetry, i.e. by projecting the equations on states of given quantum numbers, diagonalizing, in such a way, the system in block structures characterized by conserved quantum numbers.

The full set of evolution equations are computed working with gauge eigenstates which simplify systematically the evaluation in the high energy regime. We do this by giving in the appendix a complete classification of possible asymptotic states according to their isospin and CP properties. Then, scalar equations are obtained by a method we introduce here, which consists in performing appropriate traces with respect to isospin leg indices. The final equations for scalar structure functions $\underset{i}{f}_{(T,Y;CP)}$ are given in (C.11)–(C.27).

JHEP11(2005)022

Since the overall procedure is quite complicated, in section 2 we discuss the simple case of left fermions in the initial state and we show how the block diagonalization procedure helps for the numerical and practical evaluation of the evolution equations.

2. A working example: left fermions in the $g' \to 0$ limit

In this section we consider lepton initiated Drell Yan process of type $e^+(p_1)\ e(p_2) \to q(k_1)\bar{q}(k_2) + X$[1] where the two initial leptons are left handed fermions , $s = 2p_1 \cdot p_2$ is the total invariant mass and $Q^2 = 2k_1 \cdot k_2$ is the hard scale. We consider double and single log corrections in relation to the SU(2) electroweak gauge group, i.e. we work in the limit where the U(1) coupling g' is zero. This process has been analyzed in [18, 14], to which we refer for details; we consider it here for convenience and in order to establish our notations. The general formalism used to study electroweak evolution equations for inclusive observables has been set up in [11]; we summarize it here briefly. To begin with, by arguments of unitarity, final state radiation can be neglected when considering inclusive cross sections [18]. Then we are led to consider the dressing of the overlap matrix $\mathcal{O}^{\alpha'\beta'}_{\alpha\beta} = \langle\beta\beta'|S^+\ S|\alpha\alpha'\rangle$, S being the S-matrix, where only initial states indices appear explicitly (see figure 1).

At the leading level, all order resummation in the soft-collinear region is obtained by a simple expression that involves the t-channel total isospin T that couples indices α, β:

$$\mathcal{O}^H \to \mathcal{O}^{\text{resummed}} = e^{-\frac{\alpha_W}{4\pi}[T(T+1)]\log^2\frac{s}{M_w^2}}\ \mathcal{O}^H \tag{2.1}$$

α_W being the weak coupling, $\mathcal{O}_H$ the hard overlap matrix written in terms of the tree level S-matrix and $M_w \sim M_z$.

At subleading order, the dressing by soft and/or collinear radiation is described at all orders by infrared evolution equations, that are T-diagonal as far as fermions and transverse gauge bosons are concerned [11]. In order to write down the evolution equations for the case of initial left fermions, we first consider one loop corrections. At the one loop level, virtual and real corrections in NLL approximation can be written as:

$$\begin{aligned}\delta \underset{L}{\mathcal{O}}_{\alpha\beta} = \frac{\alpha_W}{2\pi}\int_{M^2}^{s}\frac{dk_\perp^2}{k_\perp^2}\int_0^1\frac{dz}{z}\Bigg\{P^R_{ff}(z)\theta\left(1-z-\frac{k_\perp}{\sqrt{s}}\right)t^A_{\beta\beta'}t^A_{\alpha'\alpha}\,\underset{L}{\mathcal{O}}^H_{\alpha'\beta'}(zp)+ \\ +P^R_{gf}(z)[t^Bt^A]_{\beta\alpha}\,\underset{g}{\mathcal{O}}^H_{AB}(zp)+C_fP^V_{ff}\left(z,\frac{k_\perp}{\sqrt{s}}\right)\underset{L}{\mathcal{O}}^H_{\alpha\beta}(p)\Bigg\}\end{aligned} \tag{2.2}$$

where $P^R_{ff}(z), P^R_{gf}(z)$ and $P^V_{ff}(z,k_\perp)$ are defined in the appendix. We note that the expression above can formally be written as

$$\underset{L}{\mathcal{O}} = \underset{L}{\mathcal{O}}^H + \alpha_W\ \sum_k^{L,R,g}\ \underset{L}{\overset{k}{\mathcal{F}}}\star\underset{k}{\mathcal{O}}^H\,. \tag{2.3}$$

[1] Note that the process considered, like all others in this paper, is fully inclusive, meaning also W, Z radiation is included.

JHEP11(2005)022

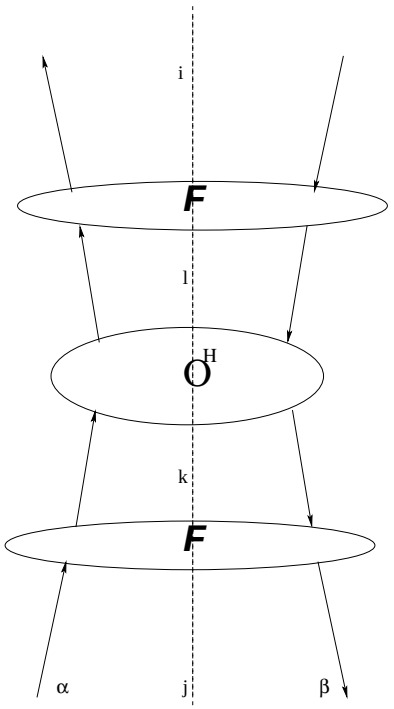

Figure 1: Graphical picture of the factorization formula.2.4

The indices below the overlap matrix label the kind of particle: $L =$ Left fermion and $g =$ gauge boson; indices α, β refer to the isospin index ($\alpha = 1$ corresponds to ν, $\alpha = 2$ to e) of the lower legs while upper legs indices are omitted.

The structure of the one loop formula (2.3) is perturbatively consistent with a general factorization formula of type (see figure 1)

$$\overset{i}{\mathcal{O}}_{j}(p_1, p_2; k_1, k_2) = \int \frac{dz_1}{z_1}\frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} \overset{k}{\mathcal{F}}_{j}(z_1; s, M^2)\, \overset{l}{\mathcal{O}}{}^{H}_{k}(z_1 p_1, z_2 p_2; k_1, k_2)\, \overset{i}{\mathcal{F}}_{l}(z_2; s, M^2) \quad (2.4)$$

where i, j label the kind of particle (L=left fermion, g=gauge boson), and where isospin flavor indices in the overlap function $\mathcal{O}$ and structure function $\mathcal{F}$ are understood (see figure 1).

If the factorization formula (2.4) is assumed to be valid at higher orders as well, the structure functions will satisfy evolution equations with respect to an infrared-collinear cutoff μ parameterizing the lowest value of $k_\perp$, as follows ($t = \log \mu^2$):

$$-\frac{\partial}{\partial t}\overset{i}{\mathcal{F}}_{j\,\alpha\beta} = \frac{\alpha_W}{2\pi}\left\{ C_f\, \overset{i}{\mathcal{F}}_{j\,\alpha\beta} \otimes P^V_{ff} + \left[t^C \overset{i}{\mathcal{F}}_{j}{}^{t}\, t^C \right]_{\alpha\beta} \otimes P^R_{ff} + \left[t^B\, t^A \right]_{\beta\alpha} \overset{i}{\mathcal{F}}_{g\,AB} \otimes P^R_{gf} \right\} \quad (2.5)$$

In these equations t^A denote the isospin matrices in the fundamental representation and $\overset{i}{\mathcal{F}}_{j\,\alpha\beta}$ denotes the distribution of a particle i (whose isospin indices are omitted) inside particle

JHEP11(2005)022

j (with isospin leg indices α, β). $\overset{i}{\underset{j}{\mathcal{F}}}{}^{t}$ is the transpose matrix $\overset{i}{\underset{j}{\mathcal{F}}}_{\beta\alpha}$. Furthermore, we have defined the convolution $[f \otimes P](x) \equiv \int_x^1 P(z) f(\frac{x}{z}) \frac{dz}{z}$; the relevant splitting functions are given in appendix. Since the index i is always kept fixed in (2.5), we will omit it from now on, with the understanding that, for instance, $\underset{j}{\mathcal{F}}$ collectively denotes all $\overset{i}{\underset{j}{\mathcal{F}}}$ with any value of i. In this sense, the evolution eqs. for $\underset{i}{\mathcal{F}}$ or $\overset{i}{\mathcal{F}}$ are analogous, in fact as we can see later, the only point where both the two indexes "i" and "j" have to be assigned simultaneously is in the boundary conditions, see eqs. (3.9), (3.10).

Eqn. (2.5) is a matricial evolution equation; in order to make it useful we can write the corresponding scalar equations. We do this by exploiting the $\mathrm{SU}(2)_L$ symmetry which allows to classify the states according to their isospin quantum numbers. We couple the lower legs α, β in figure 1 obtaining the t-channel[2] isospin eigenstates:

$$|\mathbf{T}=0\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle + |ee^*\rangle) \qquad |\mathbf{T}=1\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle - |ee^*\rangle) \tag{2.6}$$

which have $T_L^3 = 0$ since cross sections always have a given particle on leg α and its own antiparticle on leg β. We now project the structure operators $\mathcal{F}$ on these states, omitting the upper leg indices:

$$\begin{aligned} \underset{L}{f}_{(0)} &= \frac{\langle \nu\nu^* + ee^* | \underset{L}{\mathcal{F}} | \rangle}{2} = \frac{\underset{L}{\mathcal{F}}_{\nu\nu} + \underset{L}{\mathcal{F}}_{ee}}{2} = \frac{1}{2}\,\mathrm{Tr}\left[\underset{L}{\mathcal{F}}\right] \\ \underset{L}{f}_{(1)} &= \frac{\langle \nu\nu^* - ee^* | \underset{L}{\mathcal{F}} | \rangle}{2} = \frac{\underset{L}{\mathcal{F}}_{\nu\nu} - \underset{L}{\mathcal{F}}_{ee}}{2} = \mathrm{Tr}\left[t^3 \underset{L}{\mathcal{F}}\right]. \end{aligned} \tag{2.7}$$

Last step in eqs. (2.7) represents a convenient way to extract the scalar coefficients $\underset{j}{f}^{(T)}$ from $\underset{j}{\mathcal{F}}$; namely, by taking appropriate traces with respect to the soft leg j. For instance $\underset{L}{f}_{(0)}$ corresponds to $\frac{1}{2}(\underset{L}{\mathcal{F}}_{ee} + \underset{L}{\mathcal{F}}_{\nu\nu})$ [11] and can be obtained by $\mathrm{Tr}_j[\underset{j}{\mathcal{F}}]$; here and in the following the trace is taken with respect to the indices of the soft lower scale leg j . Notice that since gauge and mass eigenstates do not necessarily coincide, we have to introduce "mixed legs" with particles belonging to different gauge representations on leg α and β (more about this point, in section A). We label these cases by $i = LR$ for the mixed left/right fermion leg, $i = B3$ for the mixed $W_3 - B$ gauge bosons and $i = h3$ for the Higgs sector case. These mixing phenomena are interesting by themselves and have been considered in [14 – 16, 20] at double log level.

Projecting eq. (2.5) for instance on the $T = 0$ component we obtain:

$$-\frac{\partial}{\partial t}\,\mathrm{Tr}[\underset{f}{\mathcal{F}}] = \frac{\alpha_W}{2\pi}\left\{C_f\,\mathrm{Tr}[\underset{f}{\mathcal{F}}] \otimes P^V_{ff} + \mathrm{Tr}[t^C\,\underset{f}{\mathcal{F}}{}^{t}\,t^C] \otimes P^R_{ff} + \mathrm{Tr}[t^B\,t^A]\;\underset{g}{\mathcal{F}}_{AB}\;\otimes P^R_{gf}\right\} \tag{2.8}$$

[2]We stress that the statement "t-channel" in our case doesn't refer to the usual t-channel in the amplitude (that couples initial to final particles) but to t-channel in Overlap matrices, that are squared amplitudes (initial particles coupled to initial particles).

JHEP11(2005)022

where the traces are taken, here and in the following, with respect to the soft leg indices. This gives:

$$-\frac{\partial}{\partial t}\underset{L}{f}{}^{(0)} = \frac{\alpha_W}{2\pi}\left(\frac{3}{4}\underset{L}{f}{}^{(0)} \otimes \left(P^V_{ff} + P^R_{ff}\right) + \frac{3}{4}\underset{g}{f}{}^{(0)} \otimes P^R_{gf}\right) \tag{2.9}$$

after taking into account that $\mathrm{Tr}[t^B\ t^A]\ \underset{g}{\mathcal{F}}_{AB} = \frac{1}{2}\sum_A \underset{g}{\mathcal{F}}_{AA} = \frac{1}{2}\mathrm{Tr}[\underset{g}{\mathcal{F}}]$ and that $\underset{g}{f}{}^{(0)} = \frac{1}{3}\mathrm{Tr}[\underset{g}{\mathcal{F}}]$.

After this short introduction about the use of our projection technique, we now briefly show the utility to project on states of definite t-channel quantum number instead to have evolution equations for single particles. Following ref. [11] we can define the projections with definite value of the t-channel isospin $T = 0, 1, 2$ for fermion and gauge boson

$$\begin{aligned}
&\underset{L}{f}{}^{(1)} = \mathrm{Tr}\{t^3\,\underset{f}{\mathcal{F}}\} = \frac{f_\nu - f_e}{2}\,, && \underset{g}{f}{}^{(0)} = \frac{f_+ + f_3 + f_-}{3}\,,\\
&\underset{g}{f}{}^{(1)} = \frac{f_+ - f_-}{2}\,, && \underset{g}{f}{}^{(2)} = \frac{f_+ + f_- - 2f_3}{6}
\end{aligned} \tag{2.10}$$

where $f_e = \underset{L}{\mathcal{F}}_{ee},\ f_\nu = \underset{L}{\mathcal{F}}_{\nu\nu},\ f_+ = \underset{g}{\mathcal{F}}_{+-},\ f_- = \underset{g}{\mathcal{F}}_{-+},\ f_3 = \underset{g}{\mathcal{F}}_{33}$.

We can evaluate the evolution eqs. for the transverse gauge bosons system following the same line of the fermionic ones (see ref. [11] or next section). Then, using our projection technique described before we get the scalar equations with definite T values; precisely there are five equations coupled in three subsets characterized by the $T = 0, 1, 2$ values:

$$\text{5 factorized eqs.}\left\{\begin{array}{l}
\left\{\begin{array}{l}
-\frac{\partial}{\partial t}\underset{L}{f}{}^{(0)} = \frac{\alpha_W}{2\pi}\left(\frac{3}{4}\underset{L}{f}{}^{(0)} \otimes \left(P^R_{ff} + P^V_{ff}\right) + \frac{3}{4}\underset{g}{f}{}^{(0)} \otimes P^R_{gf}\right)\\
-\frac{\partial}{\partial t}\underset{g}{f}{}^{(0)} = \frac{\alpha_W}{2\pi}\left(2\,\underset{g}{f}{}^{(0)} \otimes \left(P^R_{gg} + P^V_{gg}\right) + \frac{1}{2}\left(\underset{L}{f}{}^{(0)} + \underset{\bar{L}}{f}{}^{(0)}\right) \otimes P^R_{fg}\right)
\end{array}\right.\\
\left\{\begin{array}{l}
-\frac{\partial}{\partial t}\underset{L}{f}{}^{(1)} = \frac{\alpha_W}{2\pi}\left(\underset{L}{f}{}^{(1)} \otimes P^V_{ff} - \frac{1}{4}\underset{L}{f}{}^{(1)} \otimes \left(P^R_{ff} + P^V_{ff}\right) + \right.\\
\qquad\qquad \left. + \frac{1}{2}\underset{g}{f}{}^{(1)} \otimes P^R_{gf}\right)\\
-\frac{\partial}{\partial t}\underset{g}{f}{}^{(1)} = \frac{\alpha_W}{2\pi}\left(\underset{g}{f}{}^{(1)} \otimes P^V_{gg} + \underset{g}{f}{}^{(1)} \otimes \left(P^R_{gg} + P^V_{gg}\right) + \right.\\
\qquad\qquad \left. + \frac{1}{2}\left(\underset{L}{f}{}^{(1)} + \underset{\bar{L}}{f}{}^{(1)}\right) \otimes P^R_{fg}\right)
\end{array}\right.\\
\left\{-\frac{\partial}{\partial t}\underset{g}{f}{}^{(2)} = \frac{\alpha_W}{2\pi}\left(3\,\underset{g}{f}{}^{(2)} \otimes P^V_{gg} - \underset{g}{f}{}^{(2)} \otimes \left(P^R_{gg} + P^V_{gg}\right)\right)\right.
\end{array}\right. \tag{2.11}$$

with similar equations holding for $\underset{\bar{L}}{f}{}^{(T)}$.

JHEP11(2005)022

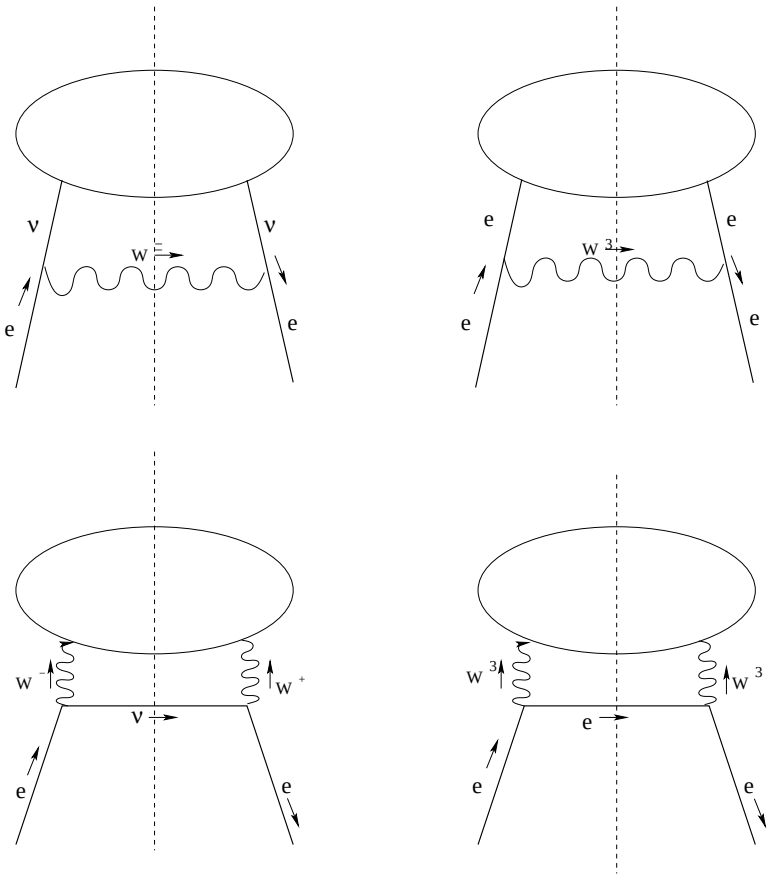

Figure 2: Graphical picture showing the evolution of the single electron component, see eqs. (2.12).

Eqs. (2.11) can be converted in evolution equations for the single components (see figure 2):

$$5 \text{ eqs.} \begin{cases} -\dfrac{4\pi}{\alpha_W}\dfrac{\partial f_\nu}{\partial t} = f_\nu \otimes (3P^V_{ff} + P^R_{ff}) + 2f_e \otimes P^R_{ff} + 2f_+ \otimes P^R_{gf} + f_3 \otimes P^R_{gf} \\ -\dfrac{4\pi}{\alpha_W}\dfrac{\partial f_e}{\partial t} = f_e \otimes (3P^V_{ff} + P^R_{ff}) + 2f_\nu \otimes P^R_{ff} + 2f_- \otimes P^R_{gf} + f_3 \otimes P^R_{gf} \\ -\dfrac{2\pi}{\alpha_W}\dfrac{\partial f_+}{\partial t} = \dfrac{f_{\bar{e}} + f_\nu}{2} \otimes P^R_{fg} + f_3 \otimes P^R_{gg} + f_+ \otimes (P^R_{gg} + 2P^V_{gg}) \\ -\dfrac{2\pi}{\alpha_W}\dfrac{\partial f_-}{\partial t} = \dfrac{f_{\bar{\nu}} + f_e}{2} \otimes P^R_{fg} + f_3 \otimes P^R_{gg} + f_- \otimes (P^R_{gg} + 2P^V_{gg}) \\ -\dfrac{2\pi}{\alpha_W}\dfrac{\partial f_3}{\partial t} = \dfrac{f_{\bar{e}} + f_e + f_{\bar{\nu}} + f_\nu}{4} \otimes P^R_{fg} + (f_- + f_+) \otimes P^R_{gg} + 2f_3 \otimes P^V_{gg} \end{cases} \tag{2.12}$$

Even if two systems (2.11), (2.12) are equivalent, eqs. (2.12) are true 5×5 system of differential equations, while the reduction to the block diagonal form of (2.11) generates a set of 2×2, 2×2 and 1×1 coupled equations.

To appreciate the block diagonalization, let us take the case of an initial fermionic parton (an electron). In this case we have only the $T = 0$ and $T = 1$ components (the fermionic system cannot couple with the $T = 2$ projection) so, while the use of

JHEP11(2005)022

eqs. (2.11) requires the solution of only the 2×2 plus 2×2 system, the use of eqs. (2.12) always forces to solve the full 5×5 set of eqs. This simplification is particularly important when the full particle spectrum of the SM is taken into account (see next section).

Looking to eqs. (2.11), it is easy to check that the leading IR log^2 come from only from the corrections proportional to P_{ii}^V present in the $T \neq 0$ components (being $P_{ii}^V \sim t$ the evolution eqs. in this case reduce to the form $\frac{\partial f_{ii}}{\partial t} \propto \alpha_W \; t \; f_{ii} \rightarrow f_{ii} \propto e^{\alpha_W \, t^2}$ and reproduce the leading behavior of eq. (2.1)) while the single logs come from the IR safe combinations $P_{ii}^V + P_{ii}^R$ or P_{ij}^R. It is interesting also to note as the IR cancellation mechanism is completely manifest in the formalism of eqs. (2.11) while in eqs. (2.12) it appears quite obscure.

Finally, the last step in obtaining the all order resummed overlap matrix, requires the evolution of the $\underset{i}{f_{(T)}}$'s according to eq. (2.11) with appropriate initial conditions, and inserting the evolved $\underset{i}{f_{(T)}}$'s into (2.4). This can by done by exploiting the recovered isospin symmetry, which allows us to write:

$$\overset{i}{\underset{j}{\mathcal{O}}}(p_1, p_2; k_1, k_2) = \sum_T \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} \overset{k}{\underset{j}{f_{(T)}}}(z_1; s, M^2) \, \overset{l}{\underset{k}{\mathcal{O}^H_{(T)}}}(z_1 p_1, z_2 p_2; k_1, k_2) \; \overset{i}{\underset{l}{f_{(T)}}}(z_2; s, M^2) \, . \tag{2.13}$$

3. Full EW evolution equations

Proceeding in analogy with previous section, we now introduce longitudinal gauge bosons and consider the full SU(2)⊗U(1) electroweak group. According to the equivalence theorem, we replace longitudinal gauge bosons with the corresponding Goldstone bosons. We choose to work in an axial gauge so that this substitution can be done without higher order corrections in the definition of the asymptotic states [21].[3]

Our notation goes as follows:

- $\underset{L_i}{\mathcal{F}_{\alpha\beta}}$ represent structure functions for left fermions of the i-family, including leptons and quarks;[4] indices $\alpha, \beta = 1, 2$ correspond to ν, e or u, d for the first family ($i = 1$) and so on. $\underset{\bar{L}_i}{\mathcal{F}_{\alpha\beta}}$ indicates the structure function for the corresponding antifermions.

- $\underset{R_i}{\mathcal{F}}$ stand for right fermions in the ith family. We work in the SM and do not consider right neutrinos. $\underset{\bar{R}_i}{\mathcal{F}}$ is for right antifermions.

[3]The authors of ref. [21] only consider double log corrections, but their statement about the absence of finite corrections in the Goldstone boson theorem is valid in the high energy limit that means for all corrections that are not mass-suppressed, and not only for double log ones. This can be seen in the derivation of their results, namely after eq. (24). The only approximation they make is to take the Sudakov limit" $k_0^2 \gg M_W^2$, which is certainly the kinematical region of interest for our results. They also state this clearly in the footnote 4.

[4]Quarks structure functions (valid for L, R or LR type) are averaged over initial color $\mathcal{F} \equiv \frac{1}{N_c} \sum_{color} \underset{quarks}{\mathcal{F}}$ with $N_c = 3$.

JHEP11(2005)022

- $\underset{LR_i}{\mathcal{F}}$ represent the "mixed legs" case where the left leg is for a left fermion and the right leg for a right fermion of the same charge for the i-family, or viceversa. Such structure functions are relevant only for the case of initial transversely polarized beams [20].

- $\underset{\phi}{\mathcal{F}}_{ab}$ represent structure functions for the Goldstone (ϕ_1, ϕ_2, ϕ_3) - Higgs (h) sector. The Goldstone modes are related to the corresponding longitudinal gauge bosons. Here $a, b = 1, 2, 3, 4$ stand for $(\phi_1, \phi_2, \phi_3, h)$.

- $\underset{g}{\mathcal{F}}_{AB}$ stand for transverse W_A gauge bosons belonging to the SU(2) sector: $A, B = 1, 2, 3$.

- $\mathcal{F}_{\mathcal{B}}$ is the structure function for the U(1) $\mathcal{B}$ gauge boson.

- $\underset{\mathcal{B}3}{\mathcal{F}}$ is the "mixed leg" case involving $\mathcal{B}, W_3$ transverse gauge bosons.

We choose to work in gauge eigenstate basis in order to represent the various structure functions. This basis is convenient for calculations since at very high energy we can consider massless propagators and therefore avoid the complications arising from mass insertions. Notice however that the asymptotic states appearing in the S matrix are truly mass eigenstates, so we need to rotate to the physical base as a final step.

The leading one loop graphs contributing, in the axial gauge, to the splitting functions are shown in figure 3. The kinematical structure is the same as in QCD; however the group structure is much more complicated due to the absence of isospin averaging on the initial states, in contrast with the corresponding (unbroken) QCD case where physical quantities are averaged over color. This implies uncanceled infrared singularities and the introduction of a new kind of infrared singular splitting functions [14].

We define $\alpha_W \equiv g^2/4\pi$, $\alpha_Y \equiv g'^2/4\pi$; the various matrices $t^A, \mathcal{T}^c, \ldots$ appearing in eqs. (3.1) are defined in the appendix. Y is the hypercharge operator, which is a diagonal matrix with appropriate eigenvalues in the different representations (left/right fermions and antifermions, longitudinal gauge bosons and so on). We now write down the infrared evolution equations for the structure functions in matrix form. Family indices here are always understood; note that the Yukawa coupling contributions proportional to the matrix $\mathcal{H}$ defined in the appendix are present only for the third family.

$$
\begin{aligned}
-\frac{\partial}{\partial t}\underset{L}{\mathcal{F}}_{\alpha\beta} = {} & \frac{\alpha_W}{2\pi}\left\{C_f\,\underset{L}{\mathcal{F}}_{\alpha\beta}\otimes P^V_{ff} + (t^C\underset{L}{\mathcal{F}}^{t}t^C)_{\beta\alpha}\otimes P^R_{ff} + \left(t^Bt^A\right)_{\beta\alpha}\underset{g}{\mathcal{F}}_{AB}\otimes P^R_{gf}\right\} + \\
& + \frac{\alpha_Y}{2\pi}\left\{\frac{1}{2}(Y^2\underset{L}{\mathcal{F}} + \underset{L}{\mathcal{F}}Y^2)_{\alpha\beta}\otimes P^V_{ff} + (Y\underset{L}{\mathcal{F}}Y)_{\alpha\beta}\otimes P^R_{ff} + (Y^2)_{\alpha\beta}\underset{\mathcal{B}}{\mathcal{F}}\otimes P^R_{gf}\right\} + \\
& + \frac{\sqrt{\alpha_W\alpha_Y}}{2\pi}\left\{(t^3Y + Yt^3)_{\alpha\beta}\underset{\mathcal{B}3}{\mathcal{F}}\otimes P^R_{gf}\right\} + \\
& + \frac{1}{32\pi^2}\Bigg\{\sum_a(\Psi^a\cdot\mathcal{H}\cdot\mathcal{H}^+\cdot\Psi^{a+})_{\alpha\alpha}\,\underset{L}{\mathcal{F}}_{\alpha\beta}\otimes\mathcal{P}^V_{LL} + \\
& \qquad + \left(\Psi^a\cdot\mathcal{H}\,\underset{R}{\mathcal{F}^t}\mathcal{H}^+\cdot\Psi^{a+}\right)_{\beta\alpha}\otimes P^R_{RL} +
\end{aligned}
$$

JHEP11(2005)022

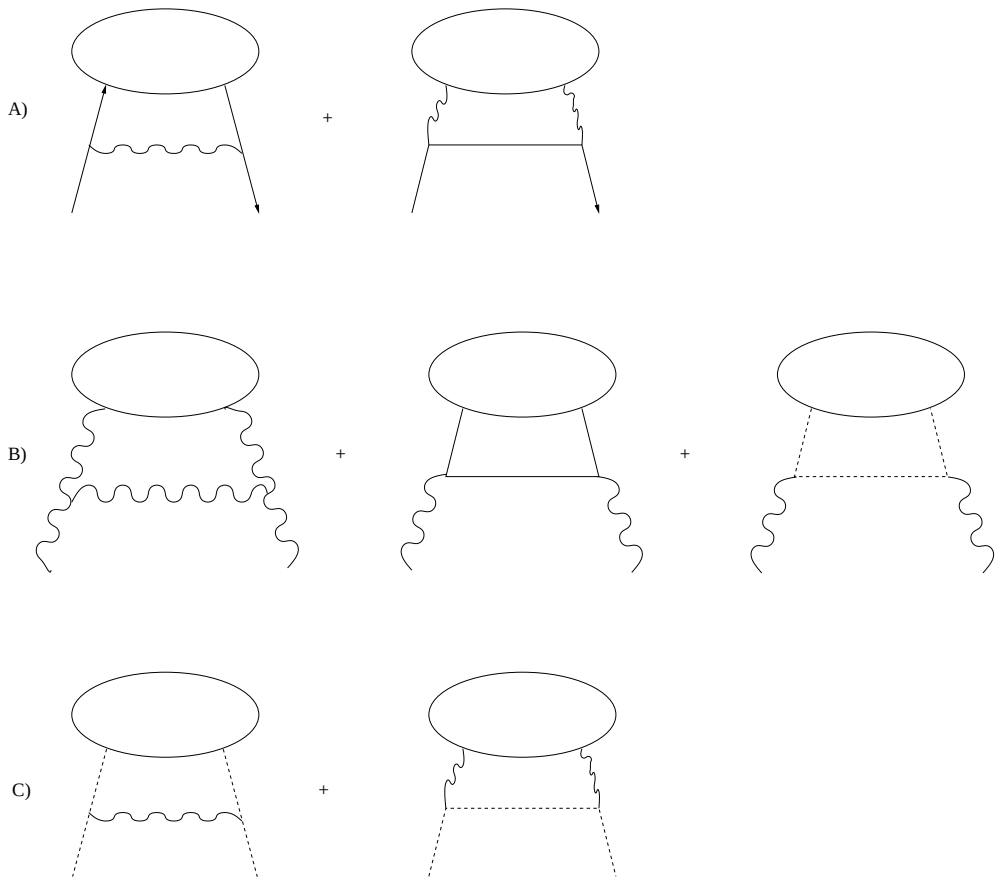

Figure 3: Leading real emission Feynman diagrams in axial gauge: (a) Feynman diagrams contributing to the evolution of the fermionic structure functions; (b) Feynman diagrams contributing to the evolution of the transverse gauge boson structure functions; (c) Feynman diagrams contributing to the evolution of the scalar structure functions. The wavy lines are transverse gauge bosons, dashed lines stay for Higgs sector particles and straight lines for fermions.

$$
\begin{aligned}
&+ (\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\beta\alpha} \underset{\phi}{\mathcal{F}}_{ab} \otimes P^R_{\phi L} \Big\};\\
-\frac{\partial}{\partial t} \underset{\bar{L}}{\mathcal{F}}_{\alpha\beta} = \frac{\alpha_W}{2\pi} &\left\{ C_f \underset{\bar{L}}{\mathcal{F}}_{\alpha\beta} \otimes P^V_{ff} + (t^C \underset{\bar{L}}{\mathcal{F}} t^C)_{\alpha\beta} \otimes P^R_{ff} + \underset{g}{\mathcal{F}}_{AB} \left(t^A t^B\right)_{\alpha\beta} \otimes P^R_{gf} \right\} +\\
&+\frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (Y^2 \underset{\bar{L}}{\mathcal{F}} + \underset{\bar{L}}{\mathcal{F}} Y^2)_{\alpha\beta} \otimes P^V_{ff} + (Y \underset{\bar{L}}{\mathcal{F}} Y)_{\alpha\beta} \otimes P^R_{ff} + (Y^2)_{\alpha\beta} \underset{\mathcal{B}}{\mathcal{F}} \otimes P^R_{gf} \right\} +\\
&+\frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ (t^3 Y + Y t^3)_{\alpha\beta} \underset{\mathcal{B}3}{\mathcal{F}} \otimes P^R_{gf} \right\} +\\
&+\frac{1}{32\pi^2} \Bigg\{ \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\alpha} \ \underset{\bar{L}}{\mathcal{F}}_{\alpha\beta} \otimes \mathcal{P}^V_{LL} +\\
&\qquad + (\Psi^a \cdot \mathcal{H} \underset{\bar{R}}{\mathcal{F}} \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\beta} \otimes P^R_{RL} +\\
&\qquad + (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+})_{\alpha\beta} \ \underset{\phi}{\mathcal{F}}_{ab} \otimes P^R_{\phi L} \Bigg\};
\end{aligned}
$$

JHEP11(2005)022

$$
\begin{aligned}
-\frac{\partial}{\partial t}\underset{R}{\mathcal{F}}_{\alpha\beta} &= \frac{\alpha_Y}{2\pi}\, y_R^2 \left\{ \underset{R}{\mathcal{F}} \otimes P^V_{ff} + \underset{R}{\mathcal{F}} \otimes P^R_{ff} + \underset{\mathcal{B}}{\mathcal{F}} \otimes P^R_{gf} \right\} + \\
&+ \frac{1}{32\pi^2} \Bigg\{ \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha}\ \underset{R}{\mathcal{F}}_{\alpha\beta} \otimes \mathcal{P}^V_{RR} + \\
&\qquad + (\mathcal{H}^+ \cdot \Psi^{a+} \underset{L}{\mathcal{F}}^t \Psi^a \cdot \mathcal{H})_{\beta\alpha} \otimes P^R_{LR} + \\
&\qquad + (\mathcal{H}^+ \cdot \Psi^{b+} \cdot \Psi^a \cdot \mathcal{H})_{\beta\alpha}\ \underset{\phi}{\mathcal{F}}_{ab} \otimes P^R_{\phi R} \Bigg\}; \\
-\frac{\partial}{\partial t}\underset{\bar{R}}{\mathcal{F}}_{\alpha\beta} &= \frac{\alpha_Y}{2\pi} y_R^2 \left\{ \underset{\bar{R}}{\mathcal{F}} \otimes P^V_{ff} + \underset{\bar{R}}{\mathcal{F}} \otimes P^R_{ff} + \underset{\mathcal{B}}{\mathcal{F}} \otimes P^R_{gf} \right\} + \\
&+ \frac{1}{32\pi^2} \Bigg\{ \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha}\ \underset{\bar{R}}{\mathcal{F}}_{\alpha\beta} \otimes \mathcal{P}^V_{RR} + \\
&\qquad + (\mathcal{H}^+ \cdot \Psi^{a+}\ \underset{\bar{L}}{\mathcal{F}} \Psi^a \cdot \mathcal{H})_{\alpha\beta} \otimes P^R_{LR} + \\
&\qquad + (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^b \cdot \mathcal{H})_{\alpha\beta}\ \underset{\phi}{\mathcal{F}}_{ab} \otimes P^R_{\phi R} \Bigg\}; \\
-\frac{\partial}{\partial t}\underset{\phi}{\mathcal{F}}_{ab} &= \frac{\alpha_W}{2\pi} \left\{ C_f\, \underset{\phi}{\mathcal{F}}_{ab} \otimes P^V_{\phi\phi} + \left(\mathcal{T}^C_L\, \underset{\phi}{\mathcal{F}}\, \mathcal{T}^C_L \right)_{ab} \otimes P^R_{\phi\phi} + \underset{g}{\mathcal{F}}_{AB} \left(\mathcal{T}^B_L \mathcal{T}^A_L \right)_{ba} \otimes P^R_{g\phi} \right\} + \\
&+ \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} \left(Y^2\, \underset{\phi}{\mathcal{F}} + \underset{\phi}{\mathcal{F}}\, Y^2 \right)_{ab} \otimes P^V_{\phi\phi} + (Y\, \underset{\phi}{\mathcal{F}}\, Y)_{ab} \otimes P^R_{\phi\phi} + (Y^2)_{ab}\, \underset{\mathcal{B}}{\mathcal{F}} \otimes P^R_{g\phi} \right\} + \\
&+ \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ (\mathcal{T}^3_L Y + Y \mathcal{T}^3_L)_{ab}\, \underset{\mathcal{B}3}{\mathcal{F}} \otimes P^R_{g\phi} \right\} + \\
&+ \frac{1}{32\pi^2} \Bigg\{ \mathrm{Tr}(\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+} + \Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+})\, \underset{\phi}{\mathcal{F}}_{ab} \otimes \mathcal{P}^V_{\phi\phi} + \\
&\qquad + \mathrm{Tr}[(\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+})\, \underset{L}{\mathcal{F}}^{\,t}] \otimes P^R_{L\phi} + \\
&\qquad + \mathrm{Tr}[(\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})\, \underset{\bar{L}}{\mathcal{F}}] \otimes P^R_{L\phi} + \\
&\qquad + \mathrm{Tr}[(\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^b \cdot \mathcal{H}) \underset{R}{\mathcal{F}}^t] \otimes P^R_{R\phi} + \\
&\qquad + \mathrm{Tr}[(\mathcal{H}^+ \cdot \Psi^{b+} \cdot \Psi^a \cdot \mathcal{H})\, \underset{\bar{R}}{\mathcal{F}}] \otimes P^R_{R\phi} \Bigg\}; \\
-\frac{\partial}{\partial t}\underset{g}{\mathcal{F}}_{AB} &= \frac{\alpha_W}{2\pi} \Bigg\{ C_g\, \underset{g}{\mathcal{F}}_{AB} \otimes P^V_{gg} + (T^C_V\, \underset{g}{\mathcal{F}}\, T^C_V)_{AB} \otimes P^R_{gg} + \\
&\qquad + \left(\sum_L \mathrm{Tr}\left[t^B\, \underset{L}{\mathcal{F}}\, {}^t t^A \right] + \sum_{\bar{L}} \mathrm{Tr}\left[t^A\, \underset{\bar{L}}{\mathcal{F}}\, t^B \right] \right) \otimes P^R_{fg} + \\
&\qquad + \mathrm{Tr}\left[\mathcal{T}^B_L \underset{\phi}{\mathcal{F}}^t \mathcal{T}^A_L \right] \otimes P^R_{\phi g} \Bigg\}; \\
-\frac{\partial}{\partial t}\underset{\mathcal{B}}{\mathcal{F}} &= \frac{\alpha_Y}{2\pi} \left\{ \underset{\mathcal{B}}{\mathcal{F}} \otimes P^R_{\mathcal{B}\mathcal{B}} + \sum_{F=L,\bar{L},R,\bar{R}} \mathrm{Tr}\left[Y\, \underset{F}{\mathcal{F}}\, Y \right] \otimes P^R_{fg} + \mathrm{Tr}\left[Y\, \underset{\phi}{\mathcal{F}}\, Y \right] \otimes P^R_{\phi g} \right\};
\end{aligned}
$$

JHEP11(2005)022

$$
\begin{aligned}
-\frac{\partial}{\partial t}\underset{\mathcal{B}3}{\mathcal{F}} &= \frac{\alpha_W}{2\pi}\left\{\frac{C_g}{2}\underset{\mathcal{B}3}{\mathcal{F}}\otimes P^V_{gg}\right\}+\\
&+\frac{\sqrt{\alpha_W\alpha_Y}}{2\pi}\left\{\sum_{F=L,\bar{L}}\mathrm{Tr}\left[Y\underset{F}{\mathcal{F}}t^3\right]\otimes P^R_{fg}+\mathrm{Tr}\left[Y\underset{\phi}{\mathcal{F}}\mathcal{T}^3_L\right]\otimes P^R_{\phi g}\right\}+\\
&+\frac{\alpha_Y}{2\pi}\left\{\frac{1}{2}\underset{\mathcal{B}3}{\mathcal{F}}\otimes P^V_{\mathcal{B}\mathcal{B}}\right\};\\
-\frac{\partial}{\partial t}\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}} &= \frac{\alpha_W}{2\pi}\left\{\frac{C_f}{2}\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}}\otimes P^V_{ff}\right\}+\\
&+\frac{\alpha_Y}{2\pi}\left\{\frac{1}{2}(y_L^2+y_R^2)\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}}\otimes P^V_{ff}+y_Ly_R\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}}\otimes P^R_{ff}\right\}+\\
&+\frac{1}{32\pi^2}\left\{\frac{1}{2}\sum_a(\Psi^a\cdot\mathcal{H}\cdot\mathcal{H}^+\cdot\Psi^{a+})_{\alpha\alpha}\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}}\otimes\mathcal{P}^V_{LL}+\right.\\
&\quad+\frac{1}{2}\sum_a(\mathcal{H}^+\cdot\Psi^{a+}\cdot\Psi^a\cdot\mathcal{H})_{\alpha\alpha}\underset{L\bar{R}}{\mathcal{F}_{\alpha\beta}}\otimes\mathcal{P}^V_{RR}+\\
&\quad\left.+\sum_a(\mathcal{H}^+\cdot\Psi^{a+}\cdot\underset{L\bar{R}}{\mathcal{F}}{}^t\cdot\mathcal{H}^\dagger\cdot\Psi^{a+})_{\beta\alpha}\otimes\mathcal{P}^R_{LR}\right\};\\
-\frac{\partial}{\partial t}\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}} &= \frac{\alpha_W}{2\pi}\left\{\frac{C_f}{2}\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}}\otimes P^V_{ff}\right\}+\\
&+\frac{\alpha_Y}{2\pi}\left\{\frac{1}{2}(y_L^2+y_R^2)\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}}\otimes P^V_{ff}+y_Ly_R\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}}\otimes P^R_{ff}\right\}+\\
&+\frac{1}{32\pi^2}\left\{\frac{1}{2}\sum_a(\Psi^a\cdot\mathcal{H}\cdot\mathcal{H}^+\cdot\Psi^{a+})_{\alpha\alpha}\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}}\otimes\mathcal{P}^V_{LL}+\right.\\
&\quad+\frac{1}{2}\sum_a(\mathcal{H}^+\cdot\Psi^{a+}\cdot\Psi^a\cdot\mathcal{H})_{\alpha\alpha}\underset{\bar{L}R}{\mathcal{F}_{\alpha\beta}}\otimes\mathcal{P}^V_{RR}+\\
&\quad\left.+\sum_a(\Psi^a\cdot\mathcal{H}\underset{\bar{L}\bar{R}}{\mathcal{F}}\cdot\Psi^a\cdot\mathcal{H})_{\alpha\beta}\otimes P^R_{LR}\right\}.
\end{aligned}
\tag{3.1}
$$

We now proceed and reduce eqs. (3.1) to scalar equations. We work in the limit of light Higgs, meaning a Higgs mass (m_H) comparable to the electroweak breaking scale (so that we can neglect terms $log(m_H^2/M^2)$). The relevant symmetry is then the (recovered) SU(2)$\otimes$ U(1) gauge group, and the states can be classified according to the total t-channel isospin $\mathbf{T}$ and the total t-channel hypercharge $\mathbf{Y}$; however we have to add the conserved quantum number CP in order to provide a complete classification of the states (see appendix). Then, our structure functions are labeled by ($\mathbf{T}, \mathbf{Y}$, CP). Furthermore, we have to consider the "mixed cases": left fermion-right fermion when we are in presence of transverse polarized initial beams [20], $B-W_3$ mixing when the asymptotic states are γ's or transverse Z [14] and $h-\phi_3$ for longitudinal Goldstone modes [15]. While we refer to the cited works for details, here we stress the fact that we work in the (ϕ_1,ϕ_2,ϕ_3,h) rather than the $1,2,\bar{1},\bar{2}$ basis used in [15]. The sum $\sum_i N_c^i$ is over all the fermions where $N_c^i=1$ for leptons and $N_c^i=3$ for quarks.

JHEP11(2005)022

The scalar evolution equations, using definitions (C.11)–(C.27) and eqs. (3.1) can finally be written as:

$$
\begin{cases}
-\dfrac{\partial}{\partial t}\underset{R_i}{f}_{(00;+)} = \dfrac{\alpha_Y}{2\pi}\left(y_R^2\,\underset{R_i}{f}_{(0,0;+)}\otimes\left(P^V_{ff}+P^R_{ff}\right)+y_R^2\,\underset{\mathcal{B}}{f}_{(0,0;+)}\otimes P^R_{gf}\right)+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}h_R^2\left(4\,\underset{R_3}{f}_{(0,0;+)}\otimes\mathcal{P}^V_{RR}+4\,\underset{L_3}{f}_{(0,0;+)}\otimes P^R_{LR}+4\,\underset{\phi}{f}_{(0,0;+)}\otimes P^R_{\phi R}\right); \\
-\dfrac{\partial}{\partial t}\underset{\mathcal{B}}{f}_{(0,0;+)} = \dfrac{\alpha_Y}{2\pi}\left(\underset{\mathcal{B}}{f}_{(0,0;+)}\otimes P^V_{\mathcal{B}\mathcal{B}}+2\sum_i N_c^i\left(2\,y_L^2\,\underset{L_i}{f}_{(0,0;+)}+\sum_R y_R^2\,\underset{R_i}{f}_{(0,0;+)}\right)\otimes P^R_{fg}+\right. \\
\qquad \left.+\,\underset{\phi}{f}_{(0,0;+)}\otimes P^R_{\phi g}\right); \\
-\dfrac{\partial}{\partial t}\underset{L_i}{f}_{(0,0;+)} = \dfrac{\alpha_W}{2\pi}\left(\dfrac{3}{4}\underset{L_i}{f}_{(0,0;+)}\otimes\left(P^V_{ff}+P^R_{ff}\right)+\dfrac{3}{4}\,\underset{g}{f}_{(0,0;+)}\otimes P^R_{gf}\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(y_L^2\,\underset{L_i}{f}_{(0,0;+)}\otimes\left(P^V_{ff}+P^R_{ff}\right)+\,y_L^2\,\underset{\mathcal{B}}{f}_{(0,0;+)}\otimes P^R_{gf}\right)+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}\left(2(h_t^2+h_b^2)\,\underset{L_3}{f}_{(0,0;+)}\otimes\mathcal{P}^V_{LL}+2\sum_R h_R^2\,\underset{R_3}{f}_{(0,0;+)}\otimes P^R_{RL}+\right. \\
\qquad \left.+\,2\,(h_t^2+h_b^2)\,\underset{\phi}{f}_{(0,0;+)}\otimes P^R_{\phi L}\right); \\
-\dfrac{\partial}{\partial t}\underset{g}{f}_{(0,0;+)} = \dfrac{\alpha_W}{2\pi}\left(2\,\underset{g}{f}_{(0,0;+)}\otimes(P^V_{gg}+P^R_{gg})+\sum_i N_c^i\,\underset{L_i}{f}_{(0,0;+)}\otimes P^R_{fg}+\underset{\phi}{f}_{(0,0;+)}\otimes P^R_{\phi g}\right); \\
-\dfrac{\partial}{\partial t}\underset{\phi}{f}_{(0,0;+)} = \dfrac{\alpha_W}{2\pi}\left(\dfrac{3}{4}\underset{\phi}{f}_{(0,0;+)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)+\dfrac{3}{4}\,\underset{g}{f}_{(0,0;+)}\otimes P^R_{g\phi}\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{4}\,\underset{\phi}{f}_{(0,0;+)}\otimes(P^R_{\phi\phi}+P^V_{\phi\phi})+\dfrac{1}{4}\,\underset{\mathcal{B}}{f}_{(0,0;+)}\otimes P^R_{g\phi}\right)+ \\
\qquad \dfrac{N_c}{32\pi^2}\left(2\,(h_t^2+h_b^2)\,\underset{\phi}{f}_{(0,0;+)}\otimes\mathcal{P}^V_{\phi\phi}+2(h_t^2+h_b^2)\,\underset{L_3}{f}_{(0,0;+)}\otimes P^R_{L\phi}+\right. \\
\qquad \left.+\,2\sum_R h_R^2\,\underset{R_3}{f}_{(0,0;+)}\otimes P^R_{R\phi}\right);
\end{cases}
\tag{3.2}
$$

$$
\begin{cases}
-\dfrac{\partial}{\partial t}\underset{R_i}{f}_{(0,0;-)} = \dfrac{\alpha_Y}{2\pi}\left(y_R^2\,\underset{R_i}{f}_{(0,0;-)}\otimes\left(P^V_{ff}+P^R_{ff}\right)\right)+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}\,h_R^2\left(4\,\underset{R_3}{f}_{(0,0;-)}\otimes\mathcal{P}^V_{RR}+4\,\underset{L_3}{f}_{(0,0;-)}\otimes P^R_{LR}\pm 4\,\underset{\phi}{f}_{(0,0;-)}\otimes P^R_{\phi R}\right) \\
\qquad (+\text{for up and - for down type fermions}); \\
-\dfrac{\partial}{\partial t}\underset{L_i}{f}_{(0,0;-)} = \left(\dfrac{3}{4}\dfrac{\alpha_W}{2\pi}+y_L^2\dfrac{\alpha_Y}{2\pi}\right)\underset{L_i}{f}_{(0,0;-)}\otimes(P^V_{ff}+P^R_{ff})+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}\left(2\,(h_t^2+h_b^2)\,\underset{L_3}{f}_{(0,0;-)}\otimes\mathcal{P}^V_{LL}+2\sum_R h_R^2\,\underset{R_3}{f}_{(0,0;-)}\otimes P^R_{RL}-\right. \\
\qquad \left.-2\,(h_t^2-h_b^2)\,\underset{\phi}{f}_{(0,0;-)}\otimes P^R_{\phi L}\right); \\
-\dfrac{\partial}{\partial t}\underset{\phi}{f}_{(00;-)} = \dfrac{\alpha_W}{2\pi}\left(\dfrac{3}{4}\,\underset{\phi}{f}_{(00;-)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{4}\,\underset{\phi}{f}_{(00;-)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
\qquad +\dfrac{N_c}{32\pi^2}\left(2\,(h_t^2+h_b^2)\,\underset{\phi}{f}_{(0,0;-)}\otimes\mathcal{P}^V_{\phi\phi}-\right. \\
\qquad\qquad -2\left(h_b^2\,\underset{R_{d_3}}{f}_{(0,0;-)}-h_t^2\,\underset{R_{u_3}}{f}_{(0,0;-)}\right)\otimes P^R_{R\phi}- \\
\qquad\qquad \left.-\,2(h_t^2-h_b^2)\,\underset{L_3}{f}_{(0,0;-)}\otimes P^R_{L\phi}\right);
\end{cases}
\tag{3.3}
$$

JHEP11(2005)022

$$
\begin{cases}
-\dfrac{\partial}{\partial t}\underset{L_i}{f}_{(1,0;+)} = \dfrac{\alpha_W}{2\pi}\left(\underset{L_i}{f}_{(1,0;+)}\otimes P^V_{ff} - \dfrac{1}{4}\underset{L_i}{f}_{(1,0;+)}\otimes\left(P^V_{ff}+P^R_{ff}\right)\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(y_L^2\ \underset{L_i}{f}_{(1,0;+)}\otimes\left(P^V_{ff}+P^R_{ff}\right)\right)+ \\
\qquad +\dfrac{\sqrt{\alpha_W\alpha_Y}}{2\pi}\left(y_L\ \underset{B3}{f}_{(1,0;+)}\otimes P^R_{gf}\right)+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}\left(2\left(h_t^2+h_b^2\right)\ \underset{L_3}{f}_{(1,0;+)}\otimes\mathcal{P}^V_{LL} - 2\left(h_t^2-h_b^2\right)\ \underset{\phi}{f}_{(1,0;+)}\otimes P^R_{\phi L}\right); \\
-\dfrac{\partial}{\partial t}\underset{B3}{f}_{(1,0;+)} = \dfrac{\alpha_W}{2\pi}\left(\dfrac{1}{2}\underset{B3}{f}_{(1,0;+)}\otimes P^V_{gg}\right)+ \\
\qquad +\dfrac{\sqrt{\alpha_W\alpha_Y}}{2\pi}\left(4\,y_L\sum_i N_c^i\ \underset{L_i}{f}_{(1,0;+)}\otimes P^R_{fg}+\underset{\phi}{f}_{(1,0;+)}\otimes P^R_{\phi g}\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{2}\underset{B3}{f}_{(1,0;+)}\otimes P^V_{BB}\right); \\
-\dfrac{\partial}{\partial t}\underset{\phi}{f}_{(10;+)} = \dfrac{\alpha_W}{2\pi}\left(\underset{\phi}{f}_{(10;+)}\otimes P^V_{\phi\phi} - \dfrac{1}{4}\underset{\phi}{f}_{(10;+)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{4}\ \underset{\phi}{f}_{(1,0;+)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+\dfrac{\sqrt{\alpha_W\alpha_Y}}{2\pi}\dfrac{1}{2}\underset{B3}{f}_{(1,0;+)}\otimes P^R_{g\phi}+ \\
\qquad +\dfrac{N_c}{32\pi^2}\left(2\left(h_t^2+h_b^2\right)\ \underset{\phi}{f}_{(1,0;+)}\otimes\mathcal{P}^V_{\phi\phi}+\left(h_b^2-h_t^2\right)\underset{L_3}{f}_{(1,0;+)}\otimes P^R_{L\phi}\right);
\end{cases}
\tag{3.4}
$$

$$
\begin{cases}
-\dfrac{\partial}{\partial t}\underset{L_i}{f}_{(1,0;-)} = \dfrac{\alpha_W}{2\pi}\left(\underset{L_i}{f}_{(1,0;-)}\otimes P^V_{ff} - \dfrac{1}{4}\underset{L_i}{f}_{(1,0;-)}\otimes\left(P^V_{ff}+P^R_{ff}\right)+\dfrac{1}{2}\ \underset{g}{f}_{(1,0;-)}\otimes P^R_{gf}\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(y_L^2\ \underset{L_i}{f}_{(1,0;-)}\otimes\left(P^V_{ff}+P^R_{ff}\right)\right)+ \\
\qquad +\dfrac{\delta_{i3}}{32\pi^2}\left(h_t^2+h_b^2\right)\left(2\ \underset{L_3}{f}_{(1,0;-)}\otimes\mathcal{P}^V_{LL}+2\ \underset{\phi}{f}_{(1,0;-)}\otimes P^R_{\phi L}\right); \\
-\dfrac{\partial}{\partial t}\underset{g}{f}_{(1,0;-)} = \dfrac{\alpha_W}{2\pi}\left(\underset{g}{f}_{(1,0;-)}\otimes P^V_{gg}+\underset{g}{f}_{(1,0;-)}\otimes\left(P^V_{gg}+P^R_{gg}\right)+ \right. \\
\qquad \left. +\sum_i N_c^i\ \underset{L_i}{f}_{(1,0;-)}\otimes P^R_{fg}+\underset{\phi}{f}_{(1,0;-)}\otimes P^R_{\phi g}\right); \\
-\dfrac{\partial}{\partial t}\underset{\phi}{f}_{(10;-)} = \dfrac{\alpha_W}{2\pi}\left(\underset{\phi}{f}_{(10;-)}\otimes P^V_{\phi\phi} - \dfrac{1}{4}\ \underset{\phi}{f}_{(1,0;-)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)+\dfrac{1}{2}\ \underset{g}{f}_{(1,0;-1)}\otimes P^R_{g\phi}\right)+ \\
\qquad +\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{4}\ \underset{\phi}{f}_{(10;-)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
\qquad +\dfrac{N_c}{32\pi^2}\left(2\left(h_t^2+h_b^2\right)\ \underset{\phi}{f}_{(1,0;-)}\otimes\mathcal{P}^V_{\phi\phi}+2\left(h_t^2+h_b^2\right)\underset{L_3}{f}_{(1,0;-)}\otimes P^R_{L\phi}\right);
\end{cases}
\tag{3.5}
$$

$$
-\frac{\partial}{\partial t}\underset{g}{f}_{(2,0;+)} = \frac{\alpha_W}{2\pi}\left(3\ \underset{g}{f}_{(2,0;+)}\otimes P^V_{gg} - \underset{g}{f}_{(2,0;+)}\otimes\left(P^V_{gg}+P^R_{gg}\right)\right);
\tag{3.6}
$$

$$
\begin{aligned}
-\frac{\partial}{\partial t}\underset{\phi}{f}_{(11;-)} &= \frac{\alpha_W}{2\pi}\left(\underset{\phi}{f}_{(11;-)}\otimes P^V_{\phi\phi} - \frac{1}{4}\ \underset{\phi}{f}_{(11;-)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
&\quad +\frac{\alpha_Y}{2\pi}\left(\frac{1}{2}\ \underset{\phi}{f}_{(11;-)}\otimes P^V_{\phi\phi} - \frac{1}{4}\ \underset{\phi}{f}_{(11;-)}\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
&\quad +\frac{N_c}{32\pi^2}\left(2\left(h_t^2+h_b^2\right)\ \underset{\phi}{f}_{(1,1;-)}\otimes\mathcal{P}^V_{\phi\phi}\right); \\
-\frac{\partial}{\partial t}\underset{\phi}{f}_{(11;+)} &= \frac{\alpha_W}{2\pi}\left(\underset{\phi}{f}_{(11;+)}\otimes P^V_{\phi\phi} - \frac{1}{4}\ \underset{\phi}{f}_{(11;+)}\otimes\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+ \\
&\quad +\frac{\alpha_Y}{2\pi}\left(\frac{1}{2}\ \underset{\phi}{f}_{(11;+)}\otimes P^V_{\phi\phi} - \frac{1}{4}\ \underset{\phi}{f}_{(11;+)}\left(P^V_{\phi\phi}+P^R_{\phi\phi}\right)\right)+
\end{aligned}
$$

JHEP11(2005)022

$$+\frac{N_c}{32\pi^2}\left(2\,(h_t^2+h_b^2)\underset{\phi}{f_{(1,1;+)}}\otimes\mathcal{P}^V_{\phi\phi}\right); \tag{3.7}$$

$$\begin{cases}
-\dfrac{\partial}{\partial t}\underset{LR_i}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right) = \dfrac{\alpha_W}{2\pi}\left(\dfrac{3}{8}\underset{LR_i}{f_u}(\frac{1}{2},\frac{1}{4};\pm)\otimes P^V_{ff}\right)+ \\
\quad+\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{2}(y_L-y^u_R)^2\underset{LR_i}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes P^V_{ff}+\right. \\
\quad\quad\left.+\,y_L\,y^u_R\underset{LR_i}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes(P^V_{ff}+P^R_{ff})\right)+ \\
\quad+\dfrac{\delta_{i3}}{32\pi^2}\left(2\,h_t^2\underset{LR_3}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^V_{RR}+\right. \\
\quad\quad+\left(h_t^2+h_b^2\right)\underset{LR_3}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^V_{LL}- \\
\quad\quad\left.-\,2h_t\,h_b\underset{LR_3}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^R_{LR}\right); \\
-\dfrac{\partial}{\partial t}\underset{LR_i}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right) = \dfrac{\alpha_W}{2\pi}\left(\dfrac{3}{8}\underset{LR_i}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes P^V_{ff}\right)+ \\
\quad+\dfrac{\alpha_Y}{2\pi}\left(\dfrac{1}{2}(y_L-y^d_R)^2\underset{LR_i}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes P^V_{ff}+\right. \\
\quad\quad\left.+\,y_L\,y^d_R\underset{LR_i}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes(P^V_{ff}+P^R_{ff})\right)+ \\
\quad+\dfrac{\delta_{i3}}{32\pi^2}\left(2\,h_b^2\underset{LR_3}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^V_{RR}+\right. \\
\quad\quad+\left(h_t^2+h_b^2\right)\underset{LR_3}{f_d}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^V_{LL}- \\
\quad\quad\left.-\,2h_t\,h_b\underset{LR_3}{f_u}\left(\frac{1}{2},\frac{1}{4};\pm\right)\otimes\mathcal{P}^R_{LR}\right).
\end{cases} \tag{3.8}$$

By looking at the above equations we can see that only structure functions that share the same set of quantum numbers are mixed by the evolution equations.

Perturbative initial conditions are independent of the particular process considered; in fact they are set at the hard scale $\sqrt{s}$, where no soft radiation is emitted, corresponding to tree level values for $\mathcal{F}$.

We have therefore (see figure1):

$$\underset{j}{\overset{i}{\mathcal{F}}}{}^{\alpha\beta}_{\alpha'\beta'}(x,\mu=\sqrt{s})=\delta_{ij}\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta(1-x) \tag{3.9}$$

or, expressed in rotated structure functions with total isospin and hypercharge

$$\underset{i}{\overset{j}{f}}{}_{(T,Y;CP)}(x,\mu=\sqrt{s})=\delta_{ij}\;\delta(1-x) \tag{3.10}$$

as can be checked by their definition in eqs. (C.11- C.27).

Eqs. (3.1)–(3.7), together with the initial conditions (3.9)–(3.10) are the main results of this paper. They are the electroweak analogous of DGLAP equations for QCD, and allow to calculate the distribution of a particle i inside a particle j at a given scale $\mu^2 \geq M_W^2$. Only electroweak effects are treated here; a complete treatment needs to take into account also the low energy ($\mu^2 \leq M_W^2$) QED running and the QCD effects of course.

As stated above, we choose to work in a gauge basis since in the high energy limit mass insertions (but not the top Yukawa insertions one) can be neglected, and calculations

JHEP11(2005)022

are simpler than in the mass eigenstates basis. However, initial (asymptotic) states are mass eigenstates and we need to rotate to this base in order to obtain physical cross sections. This can be done as a final rotation to the mass basis, once the evolved overlap matrix $\overset{i}{\underset{j}{\mathcal{O}}}(p_1,p_2;k_1,k_2)$ in the gauge basis is obtained. We write here an example in order to clarify this procedure; an analysis at the leading double log level has already been done [15, 16, 14, 20]. We discuss the change of basis for the structure functions $\mathcal{F}$, but things work in exactly the same way for the overlap matrix.

Let us consider the case of an initial photon beam ($|\gamma\rangle = c_W|\mathcal{B}\rangle + s_W|W^3\rangle$). The structure function in the mass eigenstate basis can be written in terms of structure functions of the corresponding gauge eigenstates:

$$\underset{g}{\mathcal{F}}_{\gamma\gamma} = s_W^2 \underset{g}{\mathcal{F}}_{33} + 2s_W c_W \underset{\mathcal{B}3}{\mathcal{F}} + c_W^2 \underset{\mathcal{B}}{\mathcal{F}} = s_W^2 \underset{g}{f}_{(0,0;+)} + c_W^2 \underset{\mathcal{B}}{f}_{(0,0;+)} + 2s_W c_W \underset{\mathcal{B}3}{f}_{(1,0;+)} - \frac{4}{3} s_W^2 \underset{g}{f}_{(2,0;+)} \tag{3.11}$$

obtained expanding the direct product $(c_W|\mathcal{B}\rangle + s_W|W^3\rangle) \otimes (c_W|\mathcal{B}\rangle + s_W|W^3\rangle)$, and where we used (C.11)–(C.27) to write the $\mathcal{F}$ in terms of $f_{(T,Y;CP)}$.

Acknowledgments

We are grateful to Marcello Ciafaloni for discussions and participation to the first part of our results.

A. Kernels of the evolution equations

$$P^V_{ff}(z,k_\perp) = -\delta(1-z)\left(\log\frac{s}{k_\perp^2} - \frac{3}{2}\right); \qquad P^R_{ff}(z) = \frac{1+z^2}{1-z}; \tag{A.1}$$

$$P^V_{\phi\phi}(z,k_\perp) = -\delta(1-z)\left(\log\frac{s}{k_\perp^2} - 2\right); \qquad P^R_{\phi\phi}(z) = 2\frac{z}{1-z}; \tag{A.2}$$

$$P^V_{gg}(z,k_\perp) = -\delta(1-z)\left(\log\frac{s}{k_\perp^2} - \left(\frac{11}{6} - \frac{n_f}{6} - \frac{n_s}{24}\right)\right);$$

$$P^R_{gg}(z) = 2\left(z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right);$$

$$P^V_{\mathcal{B}\mathcal{B}}(z) = -\left(2\,n_f(2y_L^2 + y_{R_u}^2 + y_{R_d}^2) + y_\phi^2\right)\delta(1-z)\frac{1}{3} \tag{A.3}$$

$$P^R_{gf}(z) = \frac{1+(1-z)^2}{z}, \qquad P^R_{g\phi}(z) = 2\frac{(1-z)}{z} \tag{A.4}$$

$$P^R_{fg}(z) = z^2 + (1-z)^2; \qquad P^R_{\phi g}(z) = z(1-z) \tag{A.5}$$

$$\begin{aligned} \mathcal{P}^V_{LL}(z) &= -\delta(1-z)\frac{1}{2}; & \mathcal{P}^V_{RR}(z) &= -\delta(1-z)\,\frac{1}{2}; \\ \mathcal{P}^V_{\phi\phi}(z) &= -\delta(1-z)\,; c & P^R_{LR}(z) &= P^R_{RL}(z) = (1-z) \\ P^R_{L\phi}(z) &= P^R_{R\phi}(z) = 1; & P^R_{\phi L}(z) &= P^R_{\phi R}(z) = z \end{aligned} \tag{A.6}$$

where $n_f = \sum_i N_c^i$ is the sum over all the fermion families ($N_c^i = 1$ for leptons and $N_c^i = 3$ for quarks), $n_s = 1$ is the number of Higgs doublets and $N_c = 3$ is a color factor.

JHEP11(2005)022

B. Fermion-scalar sector Yukawa interactions

Parametrizing the scalar fields in the following way

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}(h + i\tau_a\phi_a) = \begin{pmatrix} \phi_0 & i\phi_+ \\ i\phi_- & \phi_0^* \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} h + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & h - i\phi_3 \end{pmatrix} \tag{B.1}$$

the interaction between the scalar sector and the Left-Right fermions can be written as

$$\bar{Q}_L\, \mathbf{\Phi}\, \mathcal{H}\, Q_R + \bar{Q}_R\, \mathcal{H}^+\, \mathbf{\Phi}^+\, Q_L = \sum_{a=1}^{4} \phi^a \left(\bar{Q}_L\, \Psi^a \cdot \mathcal{H}\, Q_R + \bar{Q}_R\, \mathcal{H}^+ \cdot \Psi^{a+}\, \mathbf{\Phi}^+\, Q_L\right) \tag{B.2}$$

where $\Psi^a = i\tau^a$ for a=1,2,3 and $\Psi^4 = I_{2\times 2}$ and the third family Yukawa couplings are arranged in the following matricial form

$$\mathcal{H} = \begin{pmatrix} h_t & 0 \\ 0 & h_b \end{pmatrix} \tag{B.3}$$

C. States classification

With the above parameterization (B.1), the Higgs sector of the Standard Model includes the three Goldstone modes ϕ_1, ϕ_2, ϕ_3 and the Higgs field h. It is useful to arrange these four states in the complex form of the doublet/antidoublet matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} h + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & h - i\phi_3 \end{pmatrix} = \begin{pmatrix} \bar{1} & 1 \\ \bar{2} & 2 \end{pmatrix} \tag{C.1}$$

which transforms as $\mathbf{\Phi} \to \exp[i\alpha_L^A t^A]\mathbf{\Phi}\exp[-i\alpha_R^B t^B]$ under the $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ group. The indices $1, 2, \bar{1}, \bar{2}$ here refer to the transformation properties under $\mathrm{SU}(2)_L$.

t^A and $T^A, A = 1, 2, 3$ are the $\mathrm{SU}(2)_L$ generators in the fundamental and adjoint representation.

The other matrices appearing in the text are the generators of the $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ group in the $(\phi_1, \phi_2, \phi_3, h)$ basis; their explicit form is as follows:

$$\mathcal{T}_L^A = \frac{1}{2} \begin{pmatrix} i\,\epsilon_{BAC} & -i\,\delta_{AB} \\ i\,\delta_{AC} & 0 \end{pmatrix} \qquad \mathcal{T}_R^A = \frac{1}{2} \begin{pmatrix} i\,\epsilon_{BAC} & i\,\delta_{AB} \\ -i\,\delta_{AC} & 0 \end{pmatrix} \tag{C.2}$$

Furthermore, $(\mathcal{T}_h^3)_{ab} = i(\delta_{4a}\delta_{3b} - \delta_{3a}\delta_{4b})$ and $(\mathcal{T}_y^3)_{ab} = \delta_{3a}\delta_{4b} + \delta_{3b}\delta_{4a}$.

States in the Higgs sector can be classified according to their $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ properties, with definite $|\mathbf{T}, T_L^3; \mathbf{T}_R, T_R^3\rangle$ quantum numbers (see also appendix in [15]). However not all of the 16 possible states appear in the evolution equations. In fact, a physical cross section is always an overlap matrix element with a given particle in the left leg and its antiparticle in the right one. By charge conservation then, all the states involved in the

JHEP11(2005)022

evolution equations have t-channel charge equal to zero: $Q = T_L^3 + Y = T_L^3 + T_R^3 = 0$. Then, only 6 states are selected:

$$\begin{aligned}
|1,-1;1,1\rangle &= -|22\rangle & |1,1;1,-1\rangle &= -|\bar{1}\bar{1}\rangle \\
|1,0;1,0\rangle &= -\tfrac{|1\bar{2}\rangle+|\bar{1}2\rangle+|2\bar{1}\rangle+|\bar{2}1\rangle}{2} & |1,0;0,0\rangle &= -\frac{|1\bar{2}\rangle-|\bar{1}2\rangle+|2\bar{1}\rangle-|\bar{2}1\rangle}{2} \\
|0,0;1,0\rangle &= -\frac{|1\bar{2}\rangle+|\bar{1}2\rangle-|2\bar{1}\rangle-|\bar{2}1\rangle}{2} & |0,0;0,0\rangle &= -\frac{|1\bar{2}\rangle-|\bar{1}2\rangle-|2\bar{1}\rangle+|\bar{2}1\rangle}{2}
\end{aligned} \tag{C.3}$$

In the high energy limit we are considering the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ symmetry is recovered, and states evolve according to their total t-channel isospin $\mathbf{T}$ and hypercharge Y. However quantum numbers $|\mathbf{T}, Y\rangle$ do not provide a complete classification of the states. For instance, we have two states corresponding to $|\mathbf{T} = 0, Y = 0\rangle$: these are the states $|0,0;0,0\rangle$ and $|0,0;1,0\rangle$ appearing in (C.3). We choose to add another quantum number, CP, that acts as:[5]

$$1 \leftrightarrow -\bar{2} \quad 2 \leftrightarrow \bar{1} \quad \Rightarrow \quad \phi_1 \leftrightarrow -\phi_1 \quad \phi_2 \leftrightarrow \phi_2 \quad \phi_3 \leftrightarrow -\phi_3 \quad \phi_4 \equiv h \leftrightarrow \phi_4 \equiv h \quad \phi_+ \leftrightarrow -\phi_- \tag{C.4}$$

We can now write the states with given $|\mathbf{T}, Y^2\rangle^{(CP)}$ quantum numbers in terms of the states $|\mathbf{T}, T_L^3; \mathbf{T}_R, T_R^3\rangle$ classified according to their $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ properties and given in (C.3):

$$|0,0\rangle^{(+)} = |0,0;0,0\rangle \qquad |1,0\rangle^{(-)} = |1,0;0,0\rangle \qquad |0,0\rangle^{(-)} = |0,0,;1,0\rangle \tag{C.5}$$

$$\begin{aligned}
|1,0\rangle^{(+)} &= |1,0;1,0\rangle \qquad |1,1\rangle^{(+)} = \frac{1}{2}(|1,1;1,-1\rangle + |1,-1;1,1\rangle) \\
|1,1\rangle^{(-)} &= \frac{1}{2}(|1,1;1,-1\rangle - |1,-1;1,1\rangle)
\end{aligned} \tag{C.6}$$

It is now easy, by using (C.1), to write the above states in the $\phi_1, \phi_2, \phi_3, h$ base and to find out the corresponding f functions. For instance:

$$|0,0\rangle^{(+)} = |0,0;0,0\rangle = -\frac{|1\bar{2}\rangle-|\bar{1}2\rangle-|2\bar{1}\rangle+|\bar{2}1\rangle}{2} = |\phi_+\phi_-\rangle + |\phi_-\phi_+\rangle + |\phi_3\phi_3\rangle + |\phi_h\phi_h\rangle \tag{C.7}$$

This combination corresponds to

$$\underset{\phi}{f}_{(0,0;+)} = \frac{\underset{\phi}{\mathcal{F}}_{+-} + \underset{\phi}{\mathcal{F}}_{-+} + \underset{\phi}{\mathcal{F}}_{33} + \underset{\phi}{\mathcal{F}}_{hh}}{4}\,. \tag{C.8}$$

In the last step we have written $\underset{\phi}{f}_{(0,0;+)}$ as a trace over the lower indices of the $\mathcal{F}$ operator, in order to simplify the task of writing scalar evolution equations.

We use $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ and CP quantum numbers also to classify states in the fermions/-antifermions and gauge bosons sectors. CP acts as follows:

$$\nu \leftrightarrow \nu^* \qquad e \leftrightarrow e^* \qquad W_\mu^+ \leftrightarrow -W_\mu^- \qquad W_\mu^3 \leftrightarrow -W_\mu^3 \qquad B_\mu \leftrightarrow -B_\mu\,. \tag{C.9}$$

[5] Notice that the definition of CP given in [15] is different from the one used here.

JHEP11(2005)022

Notice that in the fermionic sector, since CP transforms a fermion in its own antifermion, the states with defined $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ properties, as defined in [15]:

$$f_L^{(0)}=\frac{f_e+f_\nu}{2}\qquad f_{\bar{L}}^{(0)}=\frac{f_{\bar{e}}+f_{\bar{\nu}}}{2}\qquad f_L^{(1)}=\frac{f_\nu-f_e}{2}\qquad f_{\bar{L}}^{(1)}=\frac{f_{\bar{e}}-f_{\bar{\nu}}}{2} \tag{C.10}$$

do not have definite CP values since CP transforms, for instance, $f_L^{(0)}$ into $f_{\bar{L}}^{(0)}$.

SU(2) symmetry acts on the gauge bosons sector as a triplet, and classifying the states does not present any particular difficulty. We can then proceed and write all the states in the fermionic, scalar, gauge bosons sectors according to their $|\mathbf{T},Y^2\rangle^{CP}$ quantum numbers. Here we write directly the corresponding structure functions $f^{(\mathbf{T},\mathbf{Y};CP)}$ and for Left fermions we write as example the case with L_i equal to left leptons of the first $i=1$ family (e and ν):

- singlet states $\mathbf{T}=0,\mathbf{Y}=0$

$$\underset{\mathcal{B}}{f}_{(0,0;+)}\equiv\underset{\mathcal{B}}{\mathcal{F}} \tag{C.11}$$

$$\underset{R_i}{f}_{(0,0;+)}\equiv\frac{1}{2}(\underset{R_i}{\mathcal{F}}+\underset{\bar{R}_i}{\mathcal{F}}) \tag{C.12}$$

$$\underset{L_i}{f}_{(0,0;+)}\equiv\frac{1}{4}\ \mathrm{Tr}\left[\underset{L_i}{\mathcal{F}}+\underset{\bar{L}_i}{\mathcal{F}}\right]=\frac{1}{4}\left(\underset{L}{\mathcal{F}}_{ee}+\underset{L}{\mathcal{F}}_{\nu\nu}+\underset{L}{\mathcal{F}}_{\bar{e}\bar{e}}+\underset{L}{\mathcal{F}}_{\bar{\nu}\bar{\nu}}\right) \tag{C.13}$$

$$\underset{\phi}{f}_{(0,0;+)}\equiv\frac{1}{4}\ \mathrm{Tr}[\underset{\phi}{\mathcal{F}}]=\frac{\underset{\phi}{\mathcal{F}}_{+-}+\underset{\phi}{\mathcal{F}}_{-+}+\underset{\phi}{\mathcal{F}}_{33}+\underset{\phi}{\mathcal{F}}_{hh}}{4} \tag{C.14}$$

$$\underset{g}{f}_{(0,0;+)}\equiv\frac{1}{3}\ \mathrm{Tr}[\underset{g}{\mathcal{F}}]=\frac{\underset{g}{\mathcal{F}}_{+-}+\underset{g}{\mathcal{F}}_{33}+\underset{g}{\mathcal{F}}_{-+}}{3} \tag{C.15}$$

$$\underset{R_i}{f}_{(0,0;-)}\equiv\frac{1}{2}(\underset{R_i}{\mathcal{F}}-\underset{\bar{R}_i}{\mathcal{F}}) \tag{C.16}$$

$$\underset{L_i}{f}_{(0,0;-)}\equiv\frac{1}{4}\ \mathrm{Tr}\left[\underset{L_i}{\mathcal{F}}-\underset{\bar{L}_i}{\mathcal{F}}\right]=\frac{1}{4}\left((\underset{L}{\mathcal{F}}_{ee}+\underset{L}{\mathcal{F}}_{\nu\nu})-(\underset{L}{\mathcal{F}}_{\bar{e}\bar{e}}+\underset{L}{\mathcal{F}}_{\bar{\nu}\bar{\nu}})\right) \tag{C.17}$$

$$\underset{\phi}{f}_{(0,0;-)}\equiv-\frac{1}{2}\ \mathrm{Tr}[\mathcal{T}_R^3\underset{\phi}{\mathcal{F}}]=\frac{\underset{\phi}{\mathcal{F}}_{+-}-\underset{\phi}{\mathcal{F}}_{-+}-i(\underset{\phi}{\mathcal{F}}_{h3}-\underset{\phi}{\mathcal{F}}_{3h})}{4} \tag{C.18}$$

- states $\mathbf{T}=1,\mathbf{Y}=0$

$$\underset{B3}{f}_{(1,0;+)}\equiv\underset{B3}{\mathcal{F}} \tag{C.19}$$

$$\underset{\phi}{f}_{(1,0;+)}\equiv\mathrm{Tr}\left[\mathcal{T}_L^3\mathcal{T}_R^3\underset{\phi}{\mathcal{F}}\right]=\frac{\underset{\phi}{\mathcal{F}}_{-+}+\underset{\phi}{\mathcal{F}}_{+-}-\underset{\phi}{\mathcal{F}}_{33}-\underset{\phi}{\mathcal{F}}_{hh}}{4} \tag{C.20}$$

$$\underset{L_i}{f}_{(1,0;+)}\equiv\frac{1}{2}\,\mathrm{Tr}\left[t^3(\underset{L_i}{\mathcal{F}}+\underset{\bar{L}_i}{\mathcal{F}})\right]=\frac{-\underset{L}{\mathcal{F}}_{ee}+\underset{L}{\mathcal{F}}_{\nu\nu}}{4}+\frac{-\underset{L}{\mathcal{F}}_{\bar{e}\bar{e}}+\underset{L}{\mathcal{F}}_{\bar{\nu}\bar{\nu}}}{4} \tag{C.21}$$

JHEP11(2005)022

$$\underset{L_i}{f}{}^{(1,0;-)} \equiv \frac{1}{2}\,\mathrm{Tr}\left[t^3(\underset{L_i}{\mathcal{F}} - \underset{\bar{L}_i}{\mathcal{F}})\right] = \frac{-\underset{L}{\mathcal{F}}_{ee} + \underset{L}{\mathcal{F}}_{\nu\nu}}{4} - \frac{-\underset{L}{\mathcal{F}}_{\bar{e}\bar{e}} + \underset{L}{\mathcal{F}}_{\bar{\nu}\bar{\nu}}}{4} \tag{C.22}$$

$$\underset{\phi}{f}{}^{(1,0;-)} \equiv -\frac{1}{2}\,\mathrm{Tr}\left[\mathcal{T}_L^3\,\underset{\phi}{\mathcal{F}}\right] = \frac{\underset{\phi}{\mathcal{F}}_{+-} - \underset{\phi}{\mathcal{F}}_{-+} + i(\underset{\phi}{\mathcal{F}}_{h3} - \underset{\phi}{\mathcal{F}}_{3h})}{4} \tag{C.23}$$

$$\underset{g}{f}{}^{(1,0;-)} \equiv -\frac{1}{2}\,\mathrm{Tr}\left[T_L^3\,\underset{g}{\mathcal{F}}\right] = \frac{\underset{g}{\mathcal{F}}_{+-} - \underset{g}{\mathcal{F}}_{-+}}{2} \tag{C.24}$$

- states $\mathbf{T} = 2, \mathbf{Y} = 0$, $CP = (+)$

$$\underset{g}{f}{}^{(2,0;+)} \equiv \frac{1}{4}\,\mathrm{Tr}\left[(3(T_L^3)^2 - 2)\,\underset{g}{\mathcal{F}}\right] = \frac{\underset{g}{\mathcal{F}}_{+-} - 2\,\underset{g}{\mathcal{F}}_{33} + \underset{g}{\mathcal{F}}_{-+}}{4} \tag{C.25}$$

- states $\mathbf{T} = 1, \mathbf{Y}^2 = 1$, $CP = (-)$

$$\underset{\phi}{f}{}^{(1,1;-)} \equiv \frac{1}{2}\,\mathrm{Tr}\left[\mathcal{T}_y^3\,\underset{\phi}{\mathcal{F}}\right] = \frac{\underset{\phi}{\mathcal{F}}_{3h} + \underset{\phi}{\mathcal{F}}_{h3}}{2} \tag{C.26}$$

- states $\mathbf{T} = 1, \mathbf{Y}^2 = 1$, $CP = (+)$

$$\underset{\phi}{f}{}^{(1,1;+)} \equiv \frac{i}{2}\,\mathrm{Tr}\left[(\mathcal{T}_y^3\mathcal{T}_h^3)\,\underset{\phi}{\mathcal{F}}\right] = \frac{\underset{\phi}{\mathcal{F}}_{33} - \underset{\phi}{\mathcal{F}}_{hh}}{2} \tag{C.27}$$

- states $\mathbf{T} = 1/2, \mathbf{Y}^2 = \frac{1}{4}$

$$\underset{LR_i}{f_u}\left(\frac{1}{2}, \frac{1}{4}; \pm\right) \equiv \frac{1}{2}\left(\underset{LR_i}{\mathcal{F}}_{11} \pm \underset{\bar{L}\bar{R}_i}{\mathcal{F}}_{11}\right) \tag{C.28}$$

$$\underset{LR_i}{f_d}\left(\frac{1}{2}, \frac{1}{4}; \pm\right) \equiv \frac{1}{2}\left(\underset{LR_i}{\mathcal{F}}_{22} \pm \underset{\bar{L}\bar{R}_i}{\mathcal{F}}_{22}\right) \tag{C.29}$$

References

[1] E. Accomando, A. Denner and S. Pozzorini, *Electroweak-correction effects in gauge boson pair production at the LHC*, *Phys. Rev.* **D 65** (2002) 073003 [hep-ph/0110114];
S. Dittmaier and M. Kramer, *Electroweak radiative corrections to W-boson production at hadron colliders*, *Phys. Rev.* **D 65** (2002) 073007 [hep-ph/0109062];
U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, *Electroweak radiative corrections to neutral-current Drell-Yan processes at hadron colliders*, *Phys. Rev.* **D 65** (2002) 033007 [hep-ph/0108274];
E. Maina, S. Moretti, M.R. Nolten and D.A. Ross, *One-loop weak corrections to the $b\bar{b}$ cross section at TeV energy hadron colliders*, *Phys. Lett.* **B 570** (2003) 205 [hep-ph/0307021];
M. Beccaria, F.M. Renard and C. Verzegnassi, *Special features of heavy quark-antiquark pair production ratios at LHC*, hep-ph/0405036;

JHEP11(2005)022

U. Baur and D. Wackeroth, *Electroweak radiative corrections to $p\overline{p} \to W^{\pm} \to l^{\pm}\nu$ beyond the pole approximation*, *Phys. Rev.* **D 70** (2004) 073015 [hep-ph/0405191];
J.H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, *Logarithmic electroweak corrections to hadronic $Z + 1$ jet production at large transverse momentum*, *Phys. Lett.* **B 609** (2005) 277 [hep-ph/0408308];
E. Accomando, A. Denner and A. Kaiser, *Logarithmic electroweak corrections to gauge-boson pair production at the LHC*, *Nucl. Phys.* **B 706** (2005) 325 [hep-ph/0409247];
W. Hollik et al., *Electroweak physics*, *Acta Phys. Polon.* **B35** (2004) 2533 hep-ph/0501246;
S. Moretti, M.R. Nolten and D.A. Ross, *Weak corrections and high E_T jets at Tevatron*, hep-ph/0503152.

[2] ECFA/DESY LC Physics Working Group collaboration, J.A. Aguilar-Saavedra et al., *Tesla technical design report part III: physics at an e^+e^- linear collider*, hep-ph/0106315.

[3] V. Berezinsky, M. Kachelriess, S. Ostapchenko. *Electroweak jet cascading in the decay of superheavy particles*, *Phys. Rev. Lett.* **89** (2002) 171802;
C. Barbot, M. Drees, *Detailed analysis of the decay spectrum of a super-heavy X particle*, *Ann. Phys. (NY)* **20** (2003) 5 hep-ph/0211406.

[4] M. Kuroda, G. Moultaka and D. Schildknecht, *Direct one loop renormalization of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ four fermion processes and running coupling constants*, *Nucl. Phys.* **B 350** (1991) 25;
G. Degrassi and A. Sirlin, *Gauge invariant selfenergies and vertex parts of the standard model in the pinch technique framework*, *Phys. Rev.* **D 46** (1992) 3104;
A. Denner, S. Dittmaier and R. Schuster, *Radiative corrections to $\gamma\gamma \to W^+W^-$ in the electroweak standard model*, *Nucl. Phys.* **B 452** (1995) 80 [hep-ph/9503442];
A. Denner and T. Hahn, *Radiative corrections to $W^+W^- \to W^+W^-$ in the electroweak standard model*, *Nucl. Phys.* **B 525** (1998) 27 [hep-ph/9711302];
W. Beenakker, A. Denner, S. Dittmaier, R. Mertig and T. Sack, *High-energy approximation for on-shell W pair production*, *Nucl. Phys.* **B 410** (1993) 245;
W. Beenakker, A. Denner, S. Dittmaier and R. Mertig, *On shell W pair production in the TeV range*, *Phys. Lett.* **B 317** (1993) 622.

[5] P. Ciafaloni and D. Comelli, *Sudakov enhancement of electroweak corrections*, *Phys. Lett.* **B 446** (1999) 278 [hep-ph/9809321].

[6] V.S. Fadin, L.N. Lipatov, A.D. Martin and M. Melles, *Resummation of double logarithms in electroweak high energy processes*, *Phys. Rev.* **D 61** (2000) 094002 [hep-ph/9910338];
M. Melles, *Subleading Sudakov logarithms in electroweak high energy processes to all orders*, *Phys. Rev.* **D 63** (2001) 034003 [hep-ph/0004056;
P. Ciafaloni and D. Comelli, *Electroweak sudakov form factors and nonfactorizable soft QED effects at nlc energies*, *Phys. Lett.* **B 476** (2000) 49 [hep-ph/9910278];
M. Melles, *Resummation of yukawa enhanced and subleading Sudakov logarithms in longitudinal gauge boson and Higgs production*, *Phys. Rev.* **D 64** (2001) 014011 [hep-ph/0012157];
M. Melles, *Electroweak renormalization group corrections in high energy processes*, *Phys. Rev.* **D 64** (2001) 054003 [hep-ph/0102097]; *Resummation of angular dependent corrections in spontaneously broken gauge theories*, *Eur. Phys. J.* **C 24** (2002) 193 [hep-ph/0108221]; *Electroweak radiative corrections in high energy processes*, *Phys. Rept.* **375** (2003) 219 [hep-ph/0104232].

JHEP11(2005)022

[7] J.H. Kuhn, A.A. Penin and V.A. Smirnov, *Summing up subleading sudakov logarithms*, *Eur. Phys. J.* **C 17** (2000) 97 [hep-ph/9912503];
J.H. Kuhn, S. Moch, A.A. Penin and V.A. Smirnov, *Next-to-next-to-leading logarithms in four-fermion electroweak processes at high energy*, *Nucl. Phys.* **B 616** (2001) 286 [hep-ph/0106298].

[8] M. Beccaria, P. Ciafaloni, D. Comelli, F.M. Renard and C. Verzegnassi, *Logarithmic expansion of electroweak corrections to four- fermion processes in the TeV region*, *Phys. Rev.* **D 61** (2000) 073005 [hep-ph/9906319]; *The role of the top mass in B production at future lepton colliders*, *Phys. Rev.* **D 61** (2000) 011301 [hep-ph/9907389];
M. Beccaria, F.M. Renard and C. Verzegnassi, *Logarithmic SUSY electroweak effects on four-fermion processes at TeV energies*, *Phys. Rev.* **D 63** (2001) 095010 [hep-ph/0007224]; *Top quark production at future lepton colliders in the asymptotic regime*, *Phys. Rev.* **D 63** (2001) 053013 [hep-ph/0010205]; *The role of universal and non universal sudakov logarithms in four fermion processes at TeV energies: the one-loop approximation revisited*, *Phys. Rev.* **D 64** (2001) 073008 [hep-ph/0103335]; *Reliability of a high energy one-loop expansion of* $e^+e^- \to W^+W^-$ *in the SM and in the MSSM*, *Nucl. Phys.* **B 663** (2003) 394 [hep-ph/0304175];
M. Beccaria, S. Prelovsek, F.M. Renard and C. Verzegnassi, *Top quark production at TeV energies as a potential SUSY detector*, *Phys. Rev.* **D 64** (2001) 053016 [hep-ph/0104245];
M. Beccaria, M. Melles, F.M. Renard and C. Verzegnassi, *SUSY scalar production in the electroweak sudakov regime of lepton colliders*, *Phys. Rev.* **D 65** (2002) 093007 [hep-ph/0112273];
M. Beccaria, M. Melles, F.M. Renard, S. Trimarchi and C. Verzegnassi, *Sudakov expansions at one loop and beyond for charged scalar and fermion pair production in SUSY models at future linear colliders*, *Int. J. Mod. Phys.* **A 18** (2003) 5069 [hep-ph/0304110]; *Charged Higgs production in the 1-TeV domain as a probe of supersymmetric models*, *Phys. Rev.* **D 68** (2003) 035014 [hep-ph/0212167];
A. Denner and S. Pozzorini, *One-loop leading logarithms in electroweak radiative corrections, I. Results*, *Eur. Phys. J.* **C 18** (2001) 461 hep-ph/0010201; *One-loop leading logarithms in electroweak radiative corrections: II. Factorization of collinear singularities*, *Eur. Phys. J.* **C 21** (2001) 63 hep-ph/0104127;
See also S. Pozzorini, *Electroweak radiative corrections at high energies*, hep-ph/0201077.

[9] M. Hori, H. Kawamura and J. Kodaira, *Electroweak Sudakov at two loop level*, *Phys. Lett.* **B 491** (2000) 275 [hep-ph/0007329];
W. Beenakker and A. Werthenbach, *New insights into the perturbative structure of electroweak sudakov logarithms: breakdown of conventional exponentiation*, *Phys. Lett.* **B 489** (2000) 148 [hep-ph/0005316]; *Electroweak two-loop Sudakov logarithms for on-shell fermions and bosons*, *Nucl. Phys.* **B 630** (2002) 3 [hep-ph/0112030];
A. Denner, M. Melles and S. Pozzorini, *Two-loop electroweak angular-dependent logarithms at high energies*, *Nucl. Phys.* **B 662** (2003) 299 [hep-ph/0301241];
U. Aglietti and R. Bonciani, *Master integrals with one massive propagator for the two- loop electroweak form factor*, *Nucl. Phys.* **B 668** (2003) 3 [hep-ph/0304028];
B. Feucht, J.H. Kuhn and S. Moch, *Fermionic and scalar corrections for the abelian form factor at two loops*, *Phys. Lett.* **B 561** (2003) 111 [hep-ph/0303016];
B. Feucht, J.H. Kuhn, A.A. Penin and V.A. Smirnov, *Two-loop Sudakov form factor in a theory with mass gap*, *Phys. Rev. Lett.* **93** (2004) 101802 [hep-ph/0404082];
A. Denner and S. Pozzorini, *An algorithm for the high-energy expansion of multi-loop*

JHEP11(2005)022

diagrams to next-to-leading logarithmic accuracy, *Nucl. Phys.* **B 717** (2005) 48 [hep-ph/0408068];
S. Pozzorini, *Next-to-leading mass singularities in two-loop electroweak singlet form factors*, *Nucl. Phys.* **B 692** (2004) 135 [hep-ph/0401087];
B. Jantzen, J.H. Kuhn, A.A. Penin and V.A. Smirnov, *Two-loop electroweak logarithms*, *Phys. Rev.* **D 72** (2005) 051301 [hep-ph/0504111].

[10] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Bloch-Nordsieck violating electroweak corrections to inclusive TeV scale hard processes*, *Phys. Rev. Lett.* **84** (2000) 4810 [hep-ph/0001142].

[11] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Towards collinear evolution equations in electroweak theory*, *Phys. Rev. Lett.* **88** (2002) 102001 [hep-ph/0111109].

[12] V.N. Gribov and L.N. Lipatov, *Deep inelastic e p scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438;
J. Kripfganz and H. Perlt, *Electroweak radiative corrections and quark mass singularities*, *Z. Physik* **C 41** (1988) 319;
H. Spiesberger, *Qed radiative corrections for parton distributions*, *Phys. Rev.* **D 52** (1995) 4936 [hep-ph/9412286];
M. Roth and S. Weinzierl, *QED corrections to the evolution of parton distributions*, *Phys. Lett.* **B 590** (2004) 190 [hep-ph/0403200].

[13] L.N. Lipatov, *The parton model and perturbation theory*, *Sov. J. Nucl. Phys.* **20** (1975) 94;
G. Altarelli, G. Parisi, *Asymptotic freedom in parton language*, *Nucl. Phys.* **B 126** (1977) 298;
Y.L. Dokshitzer, *Calculation of the structure functions for deep inelastic scattering and e^+e^- annihilation by perturbation theory in quantum chromodynamics. (in russian)*, *Sov. Phys. JETP* **46** (1977) 641.

[14] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Electroweak double logarithms in inclusive observables for a generic initial state*, *Phys. Lett.* **B 501** (2001) 216 [hep-ph/0007096].

[15] M. Ciafaloni, P. Ciafaloni, D. Comelli, *Enhanced electroweak corrections to inclusive boson fusion processes at the TeV scale*, *Nucl. Phys.* **B 613** (2001) 382 [hep-ph/0103316].

[16] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Bloch-Nordsieck violation in spontaneously broken abelian theories*, *Phys. Rev. Lett.* **87** (2001) 211802 [hep-ph/0103315].

[17] A. Denner and S. Pozzorini, *One-loop leading logarithms in electroweak radiative corrections. I: results*, *Eur. Phys. J.* **C 18** (2001) 461 [hep-ph/0010201]; *One-loop leading logarithms in electroweak radiative corrections: II. Factorization of collinear singularities*, *Eur. Phys. J.* **C 21** (2001) 63 [hep-ph/0104127].

[18] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Electroweak bloch-nordsieck violation at the TeV scale: 'strong' weak interactions?*, *Nucl. Phys.* **B 589** (2000) 359 [hep-ph/0004071].

[19] R. Doria, J. Frenkel and J.C. Taylor, *Counter example to nonabelian bloch-nordsieck theorem*, *Nucl. Phys.* **B 168** (1980) 93;
G.T. Bodwin, S.J. Brodsky, G.P. Lepage, *Initial state interactions and the Drell-Yan process*, *Phys. Rev. Lett.* **47** (1981) 1799;
A.H. Mueller, *Sudakov effects and initial state interactions in the Drell-Yan process*, *Phys. Lett.* **B 108** (1982) 355;
W.W. Lindsay, D.A. Ross and C.T. Sachrajda, *On the cancellation of long distance, nonfactorizing contributions to the Drell-Yan cross-section*, *Nucl. Phys.* **B 214** (1983) 61;
P.H. Sorensen and J.C. Taylor, *Do quarks propagate?*, *Nucl. Phys.* **B 238** (1984) 284;

JHEP11(2005)022

S. Catani, M. Ciafaloni and G. Marchesini, *Asymptotic coherent states and color screening*, *Phys. Lett.* **B 168** (1986) 284.

[20] P. Ciafaloni, D. Comelli and A. Vergine, *Sudakov electroweak effects in transversely polarized beams*, *JHEP* **07** (2004) 039 [hep-ph/0311260].

[21] W. Beenakker and A. Werthenbach, *Electroweak two-loop Sudakov logarithms for on-shell fermions and bosons*, *Nucl. Phys.* **B 630** (2002) 3 [hep-ph/0112030].

JHEP11(2005)022